



# FIFE and BOREAS Reunion



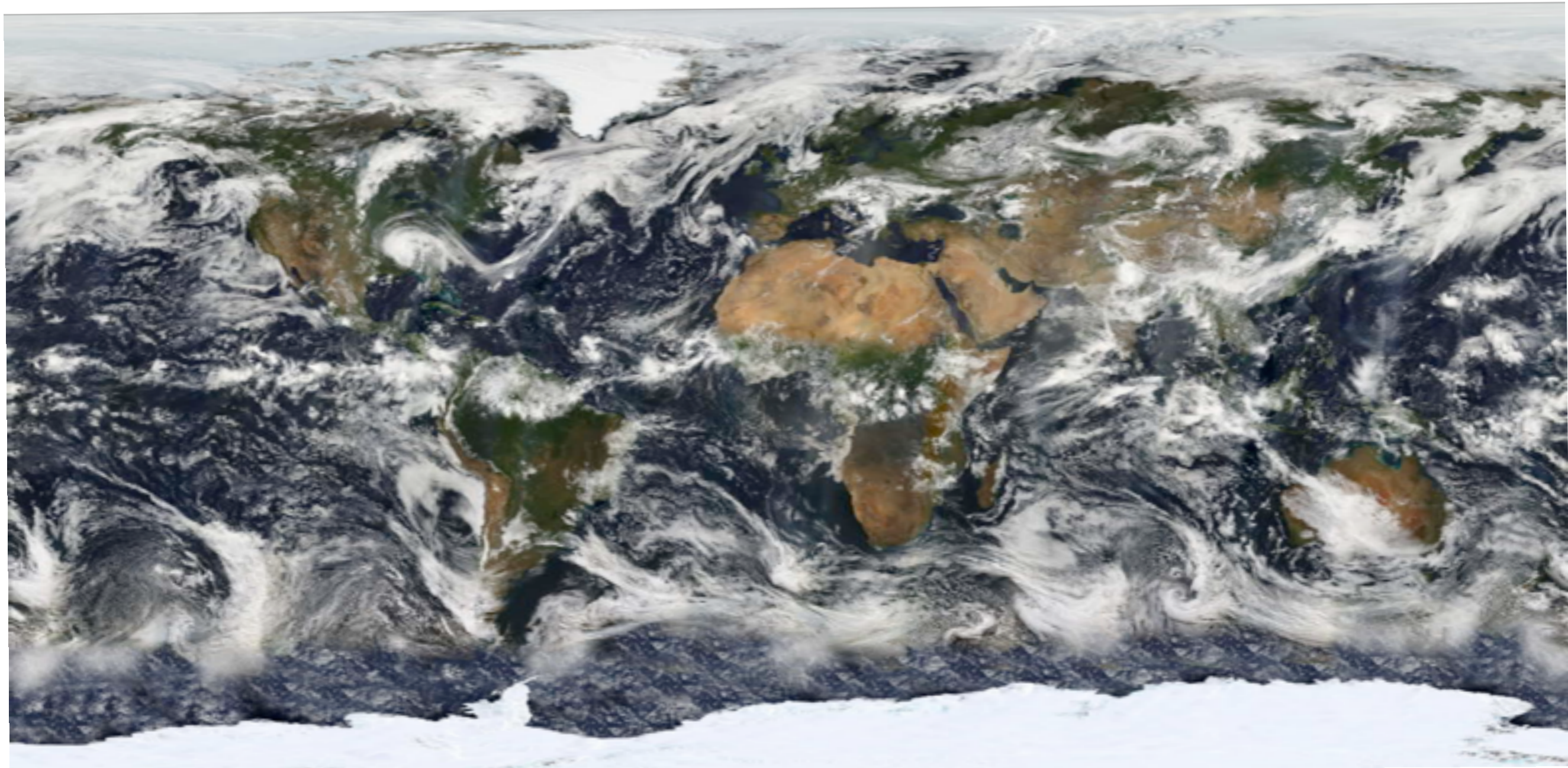
**C**ARNEGIE INSTITUTION

FOR SCIENCE

DEPARTMENT OF GLOBAL ECOLOGY



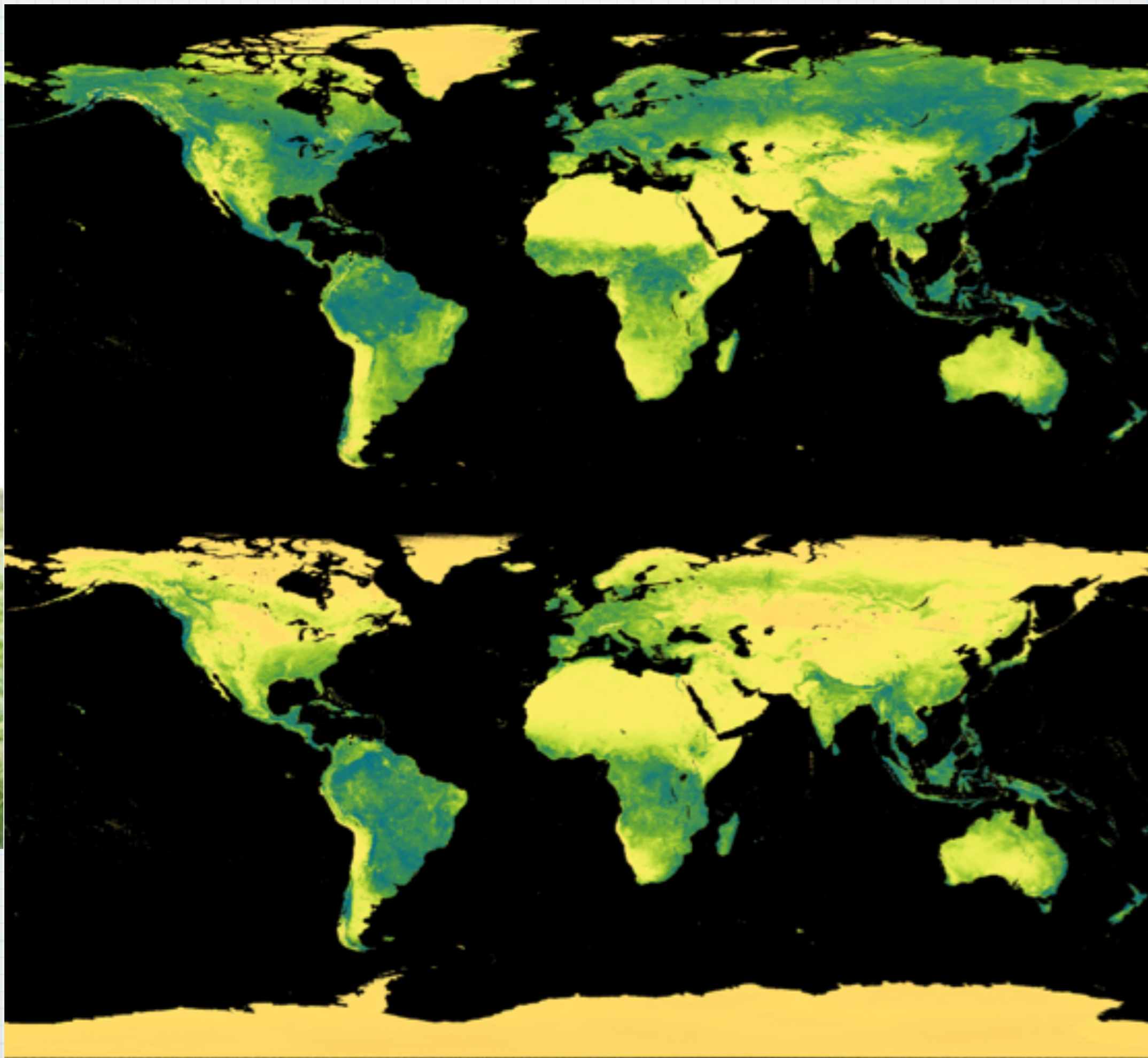
## Physiology leaf to Orbit: Joe Berry



AVHRR  
and  
NDVI



Jim Tucker



Inez Fung



# The Global Carbon Cycle

JOURNAL OF GEOPHYSICAL RESEARCH. VOL. 88, NO. C2, PAGES 1281-1294, FEBRUARY 20, 1983

## Three-Dimensional Tracer Model Study of Atmospheric CO<sub>2</sub>: Response to Seasonal Exchanges With the Terrestrial Biosphere

I. FUNG,<sup>1</sup> K. PRENTICE,<sup>2</sup> E. MATTHEWS,<sup>3</sup> J. LERNER,<sup>3</sup> AND G. RUSSELL

*Institute for Space Studies, NASA/Goddard Space Flight Center, New York, New York 10025*

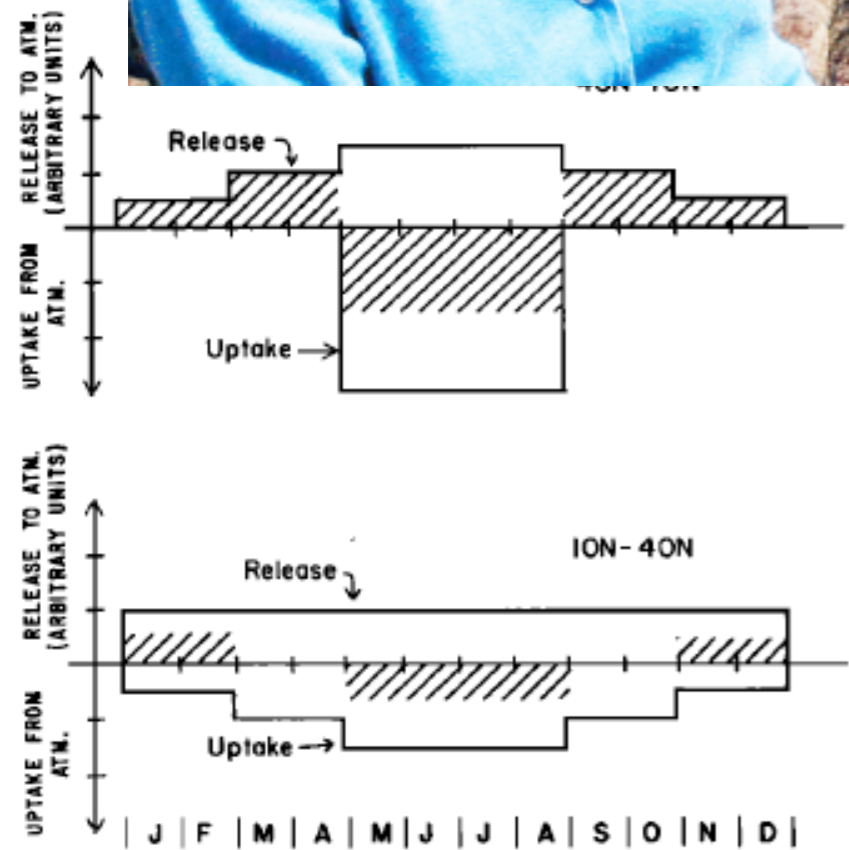


Fig. 3. Seasonality of biospheric uptake and release of CO<sub>2</sub> [after Azevedo, 1982] employed in experiment 3.

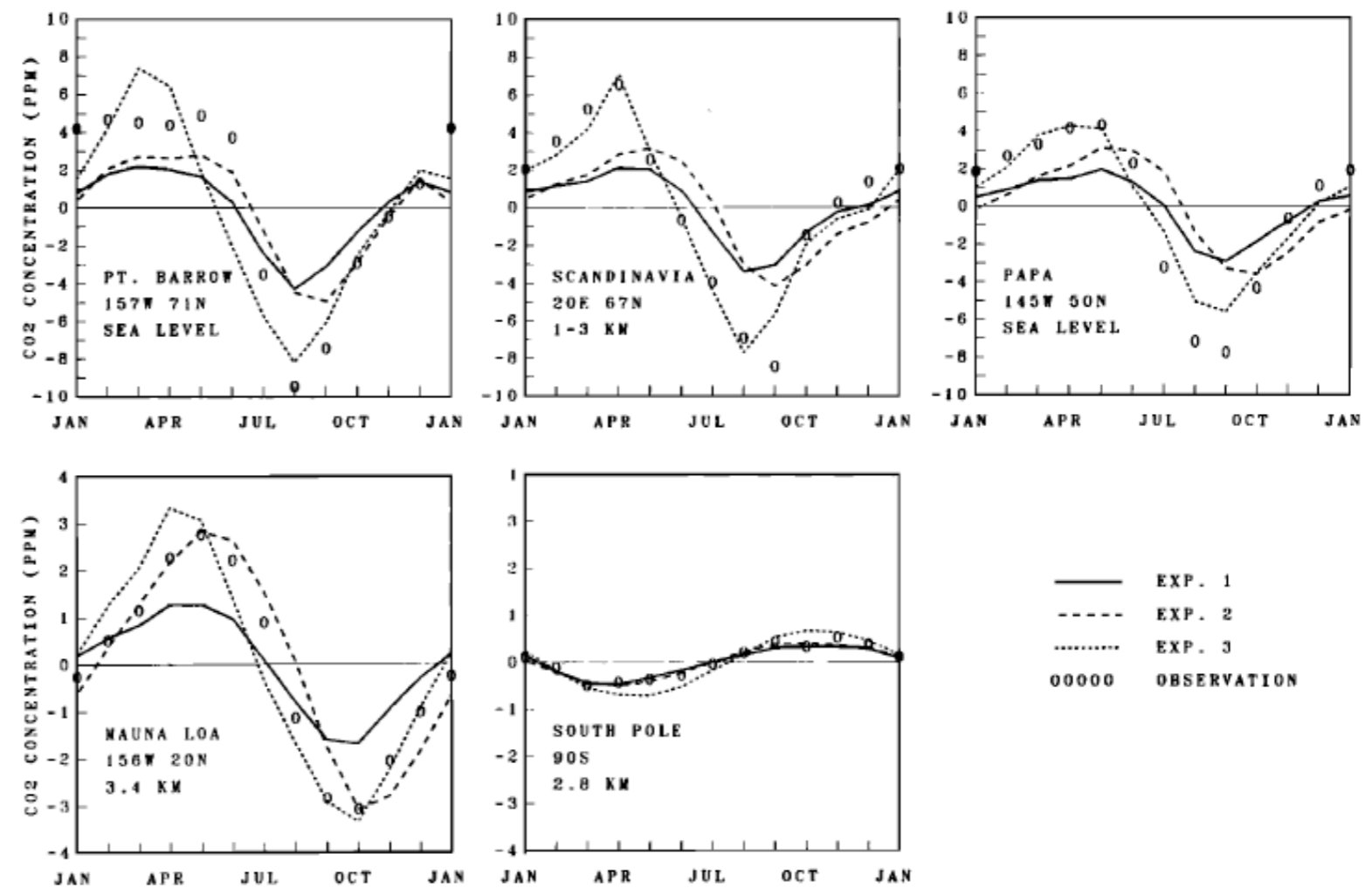


Fig. 4. Model simulated and observed annual cycles of CO<sub>2</sub> at five locations. Observations for Point Barrow, Papa, Mauna Loa, and the south pole are taken from *Pearman and Hyson [1980]* and that for Scandinavia from *Bolin and Bischof [1970]*.

# The Flying Carpet Plots

NATURE VOL. 319 16 JANUARY 1986

ARTICLES

## Relationship between atmospheric CO<sub>2</sub> variations and a satellite-derived vegetation index

C. J. Tucker\*, I. Y. Fung†, C. D. Keeling‡ & R. H. Gammon§

\* NASA/Goddard Space Flight Center, Code 623, Greenbelt, Maryland 20771, USA

† NASA/Goddard Institute for Space Studies, New York, New York 10025, USA and Lamont-Doherty Geological Observatory of Columbia University, Palisades, New York, New York 10964, USA

‡ Scripps Institution of Oceanography, La Jolla, California 92093, USA

§ NOAA/GMCC, Boulder, Colorado 80302, USA

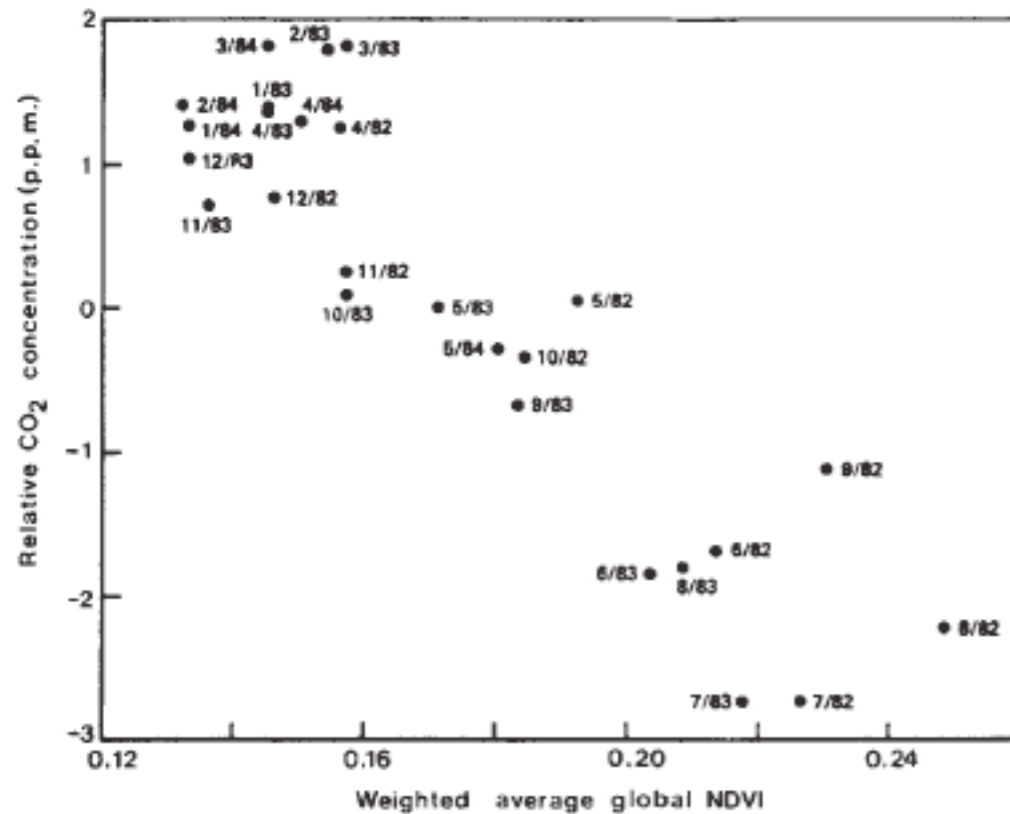


Fig. 4 The globally averaged atmospheric CO<sub>2</sub> concentration plotted against the globally averaged NDVI with a time lag of 1 month. The CO<sub>2</sub> data are from the global network of 20 NOAA/GMCC stations.

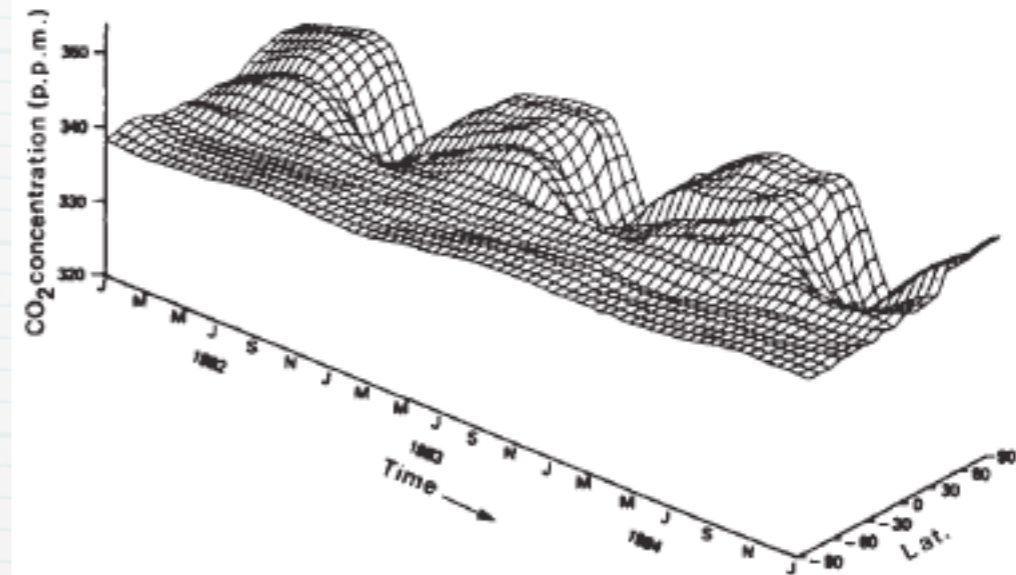
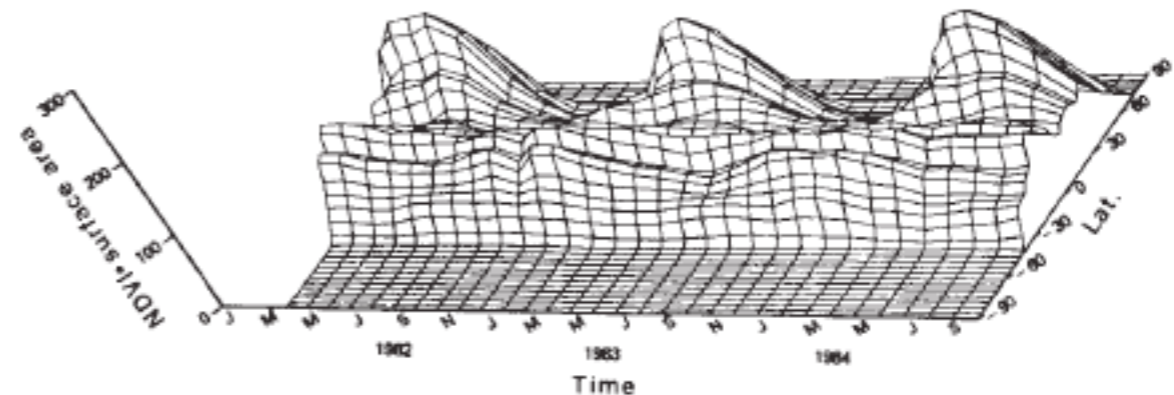


Fig. 1 Variation of global atmospheric CO<sub>2</sub> concentrations with latitude and time based on the NOAA/GMCC flask measurements for 1982-84.



# Land-Surface Modeling

## Canopy Conductance

Piers Sellers

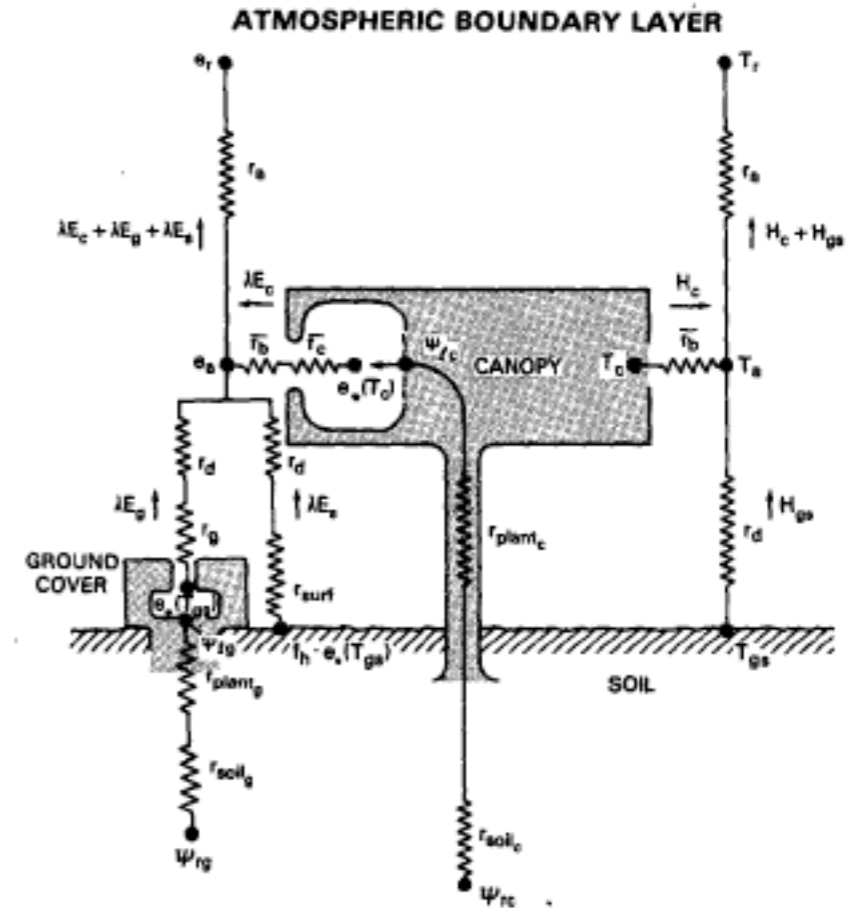


FIG. 2. Framework of the Simple Biosphere (SiB). The transfer pathways for latent and sensible heat flux are shown on the left- and right-hand sides of the diagram respectively. The treatment of radiation and intercepted water has been omitted for clarity. Symbols are defined in Table 2.

### A Simple Biosphere Model (SiB) for Use within General Circulation Models

P. J. SELLERS AND Y. MINTZ

*Dept. of Meteorology, University of Maryland, College Park, MD 20742*

Y. C. SUD

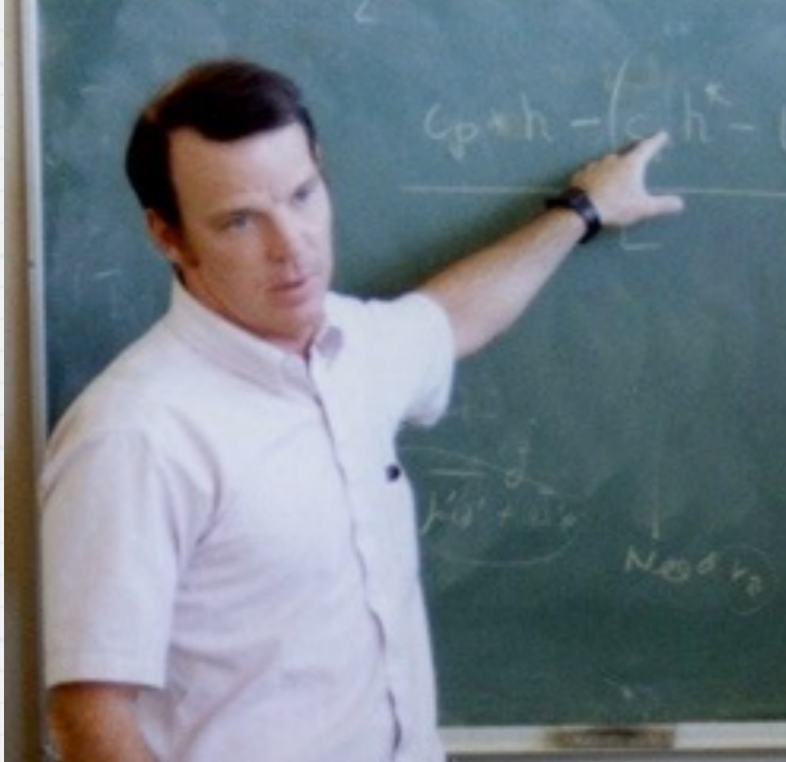
*Laboratory for Atmospheres, NASA/Goddard Space Flight Center, Greenbelt, MD 20771*

A. DALCHER

*Sigma Data Computing Corp., Rockville, MD 20850*

(Manuscript received 26 February 1985, in final form 5 September 1985)

# Dave Randall



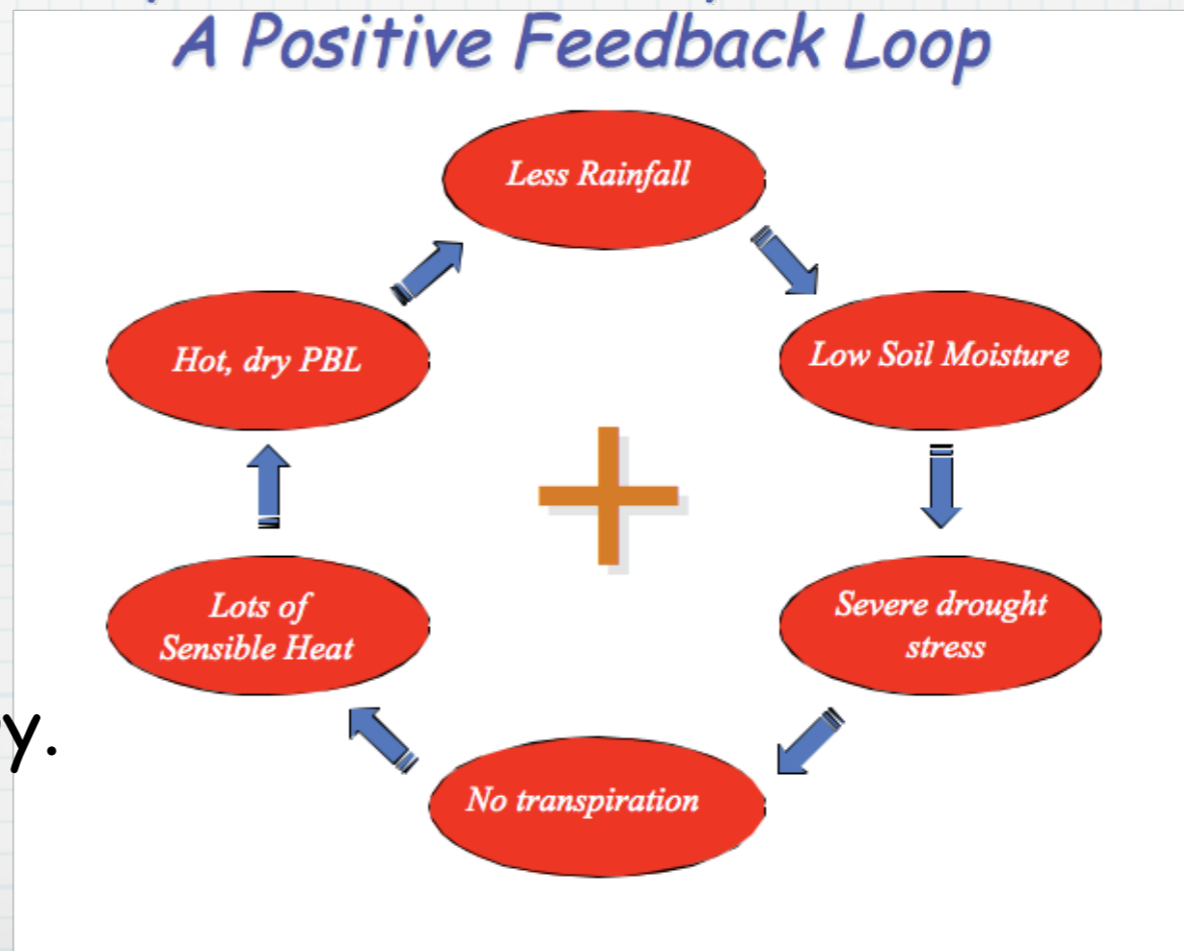
## Effects of Implementing the Simple Biosphere Model in a General Circulation Model

N. SATO,\* P. J. SELLERS, D. A. RANDALL,\*\* E. K. SCHNEIDER, J. SHUKLA,  
J. L. KINTER III, Y-T. HOU AND E. ALBERTAZZI

SiB worked fine when run with "prescribed" climate.

However, when it was run in the climate model, the land areas of the planet dried up and became deserts.

### *Precipitation and Ecosystem Stress: A Positive Feedback Loop*



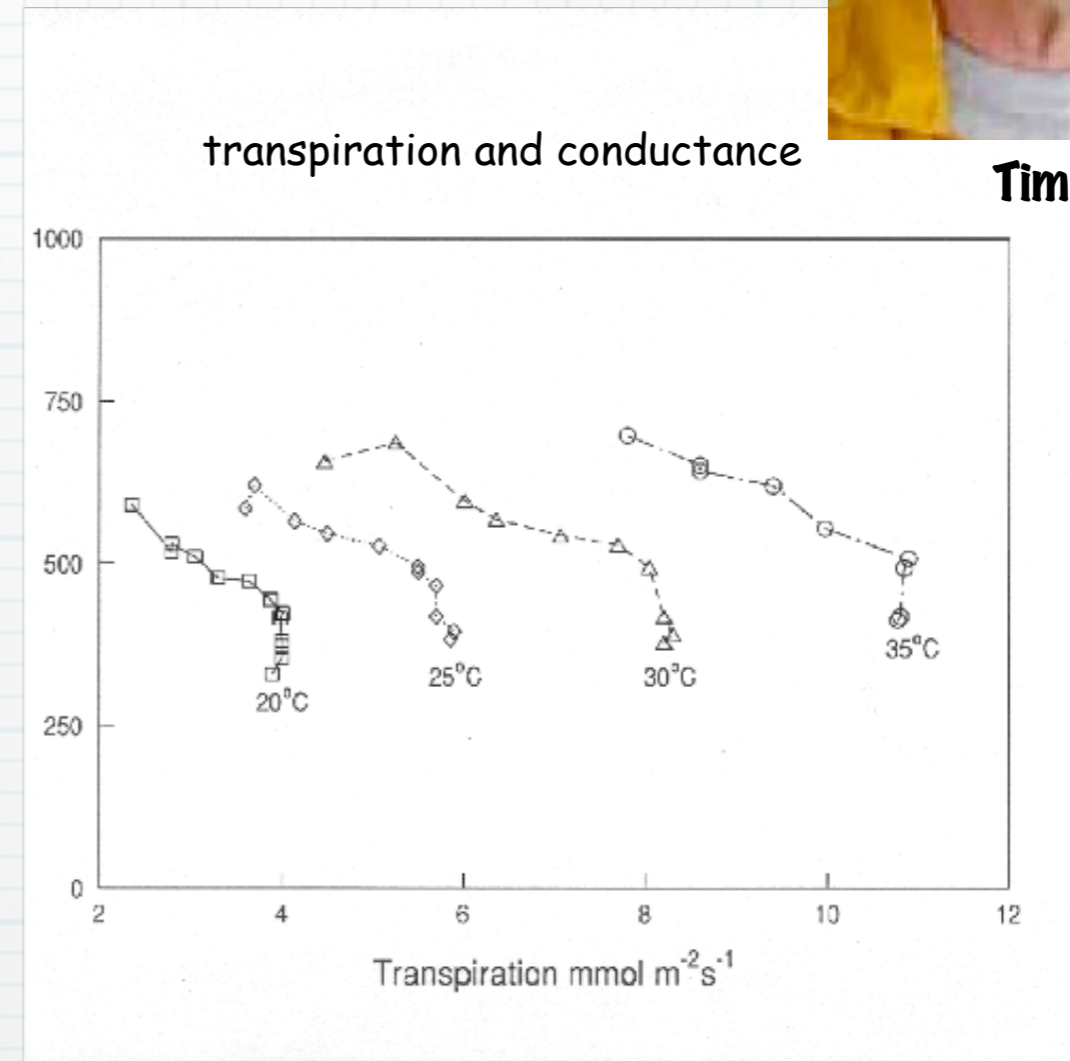
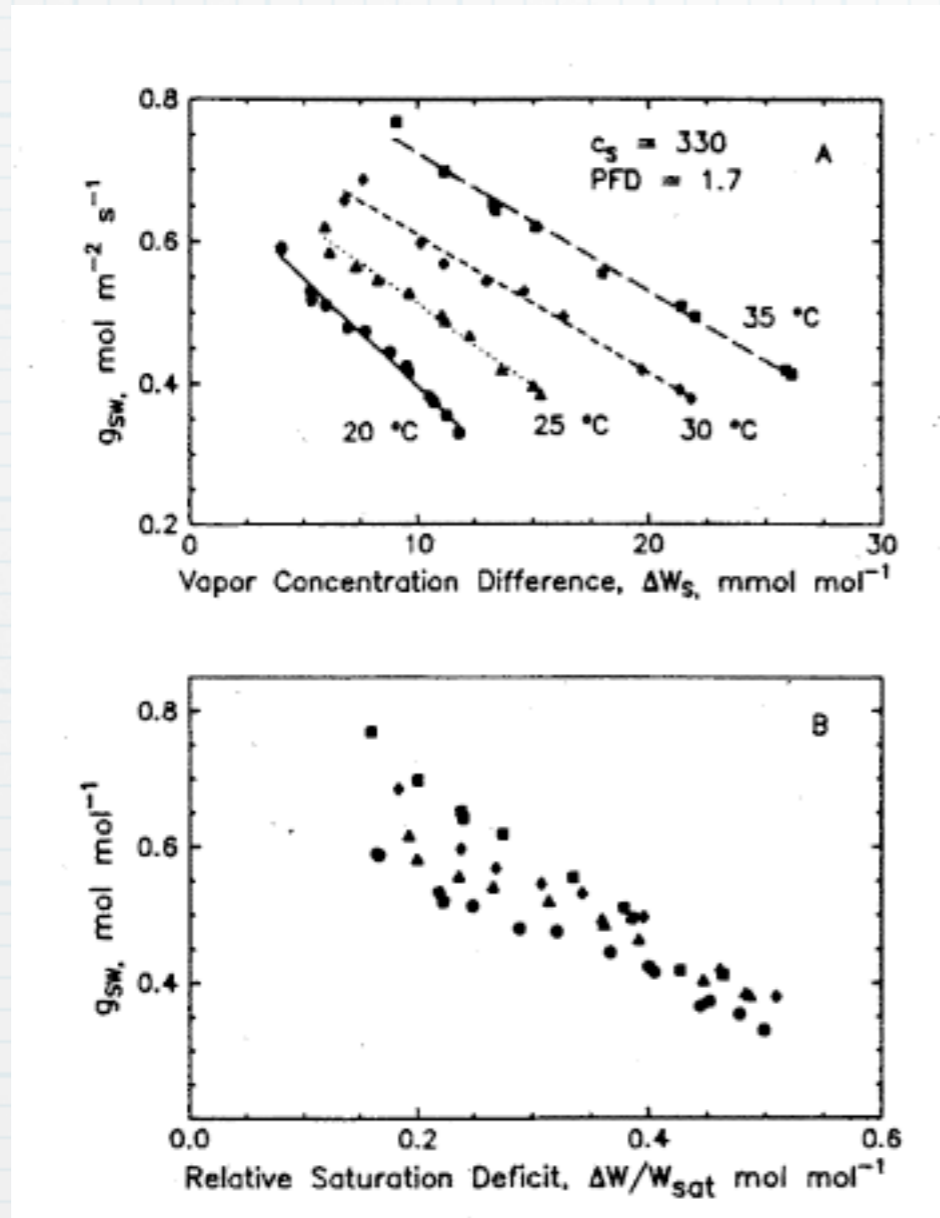
Dave referred to this as:  
"Stomatal Suicide"

This gave us an opening to  
try to fix it with physiology.

Stomatal Conductance is a linear function of VPD but the sensitivity changes with temperature.

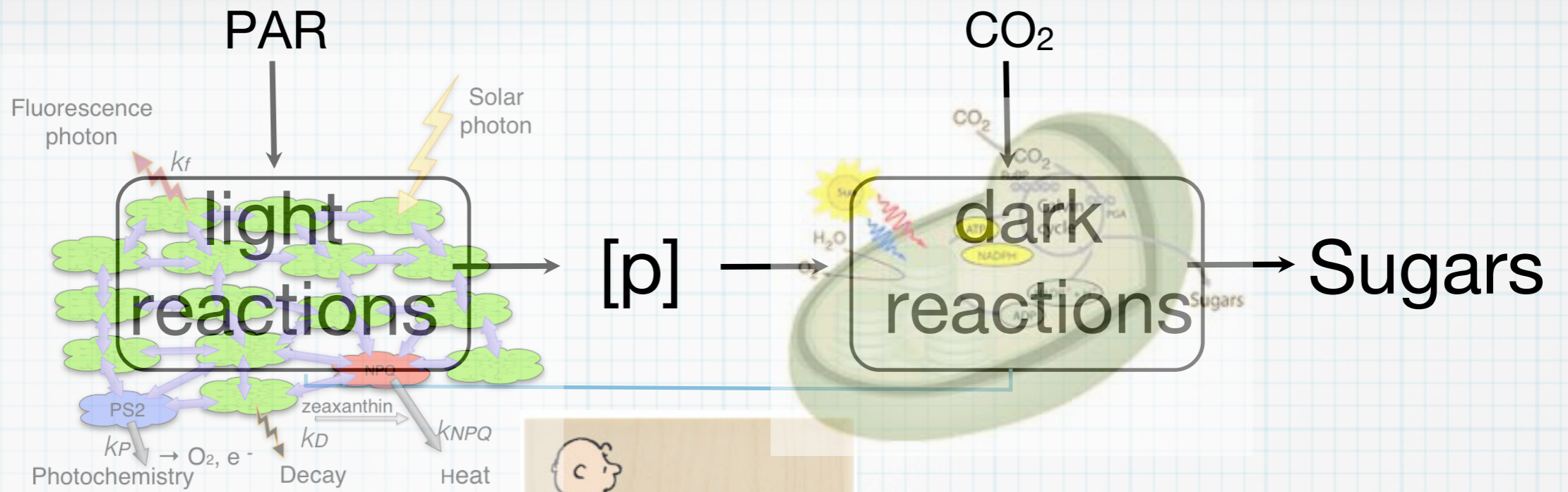


Tim Ball

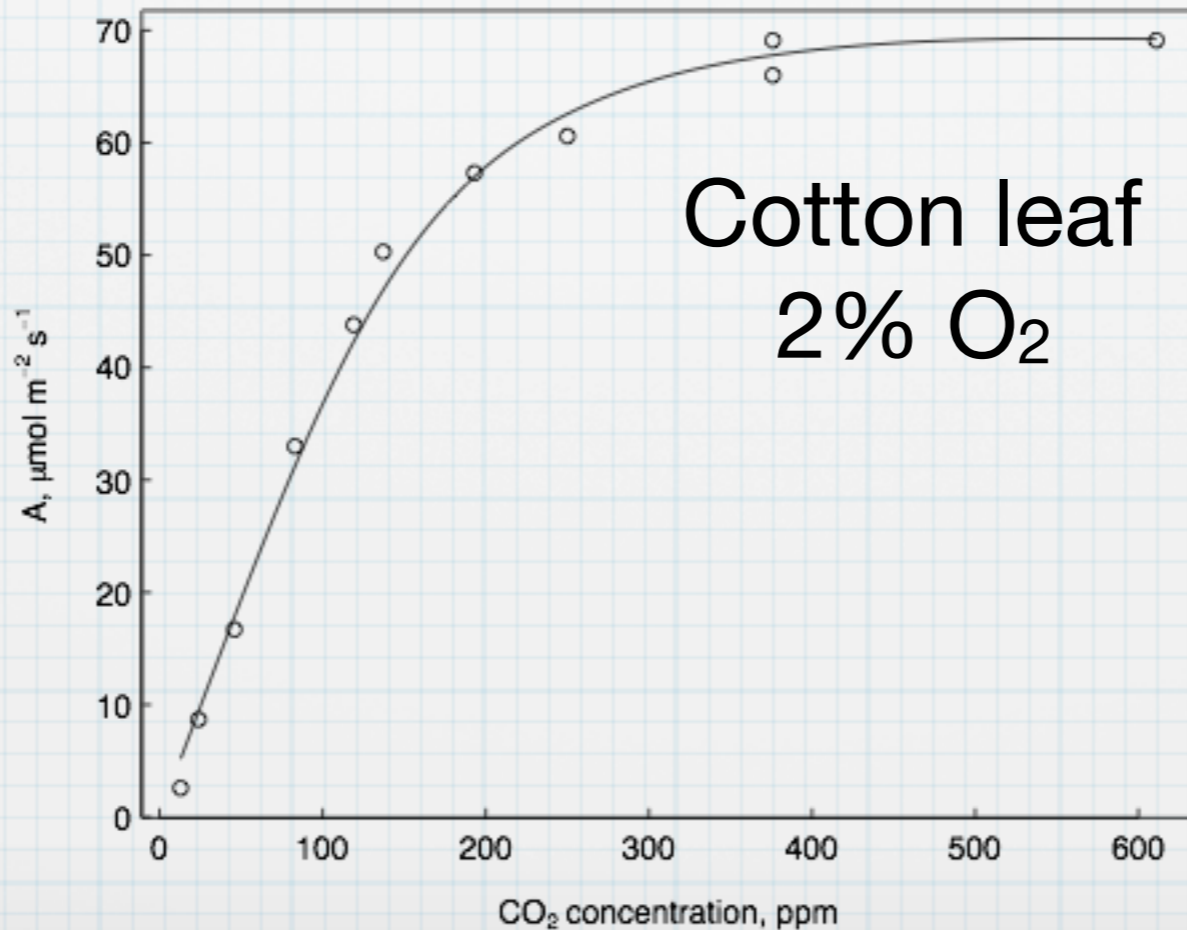


If this isn't taken into account, stomata shut when temperature increases, causing temperature to increase further.

Using our stomata-photosynthesis model "cured" stomatal depression.



Graham Farquhar



Susanne von Caemmerer





$$A \approx \min \begin{cases} J_E \\ J_C \\ J_S \end{cases}$$

C<sub>3</sub>

$$J_E = a \times \alpha \times Q_p \frac{p_i - \Gamma_*}{p_i + 2\Gamma_*}$$

$$J_C = \frac{V_m(p_i - \Gamma_*)}{p_i + K_c(1 + [O_2]/K_o)}$$

$$J_S = V_m/2$$

C<sub>4</sub>

$$J_i = a \alpha_r f Q_p$$

$$J_c = p_i \left( k_p - \frac{L}{p_i} \right) / P$$

$$J_e = V_{\max}$$

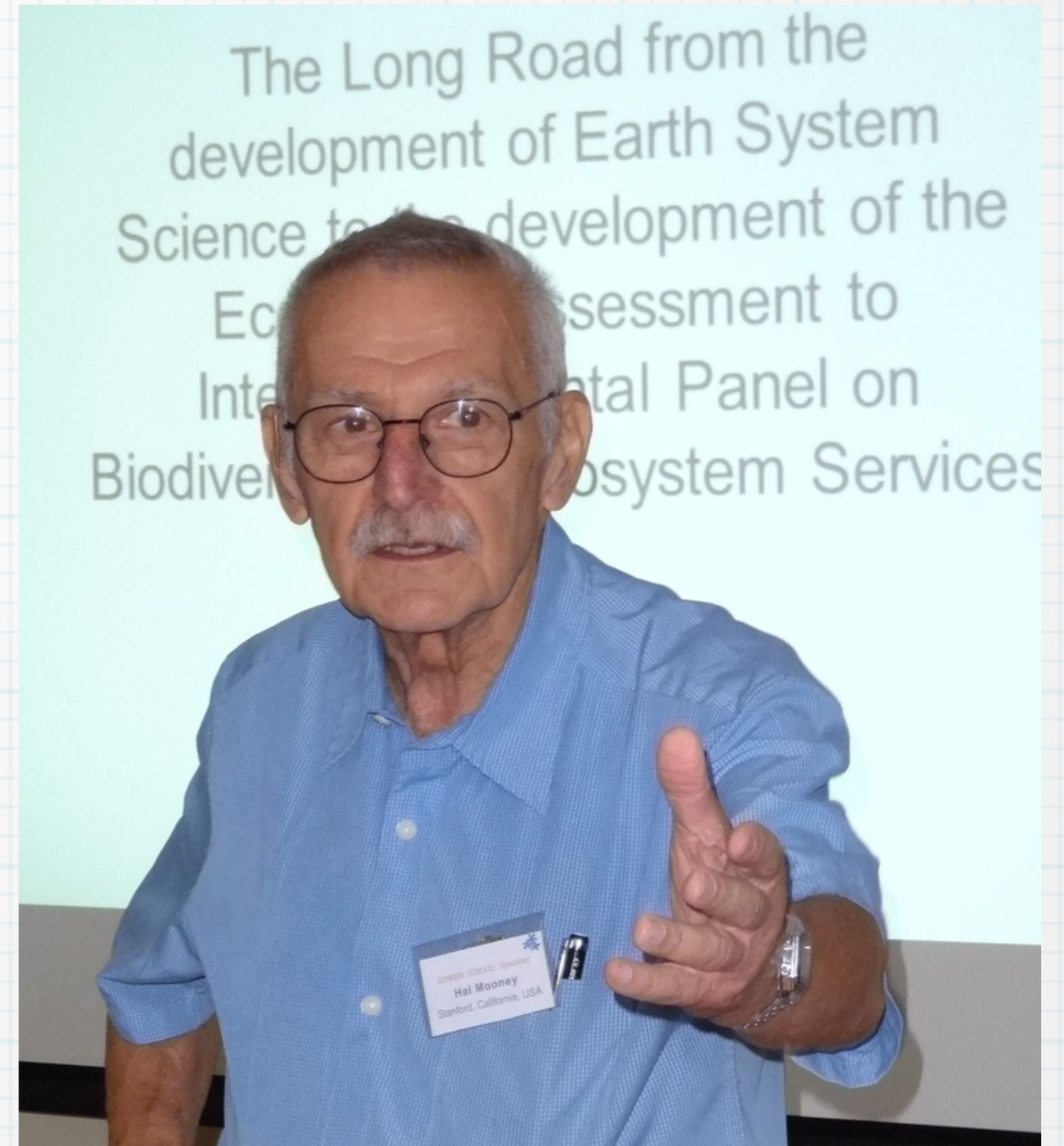
$$\theta J_P^2 - J_P(J_E + J_C) + J_E J_C = 0$$

and

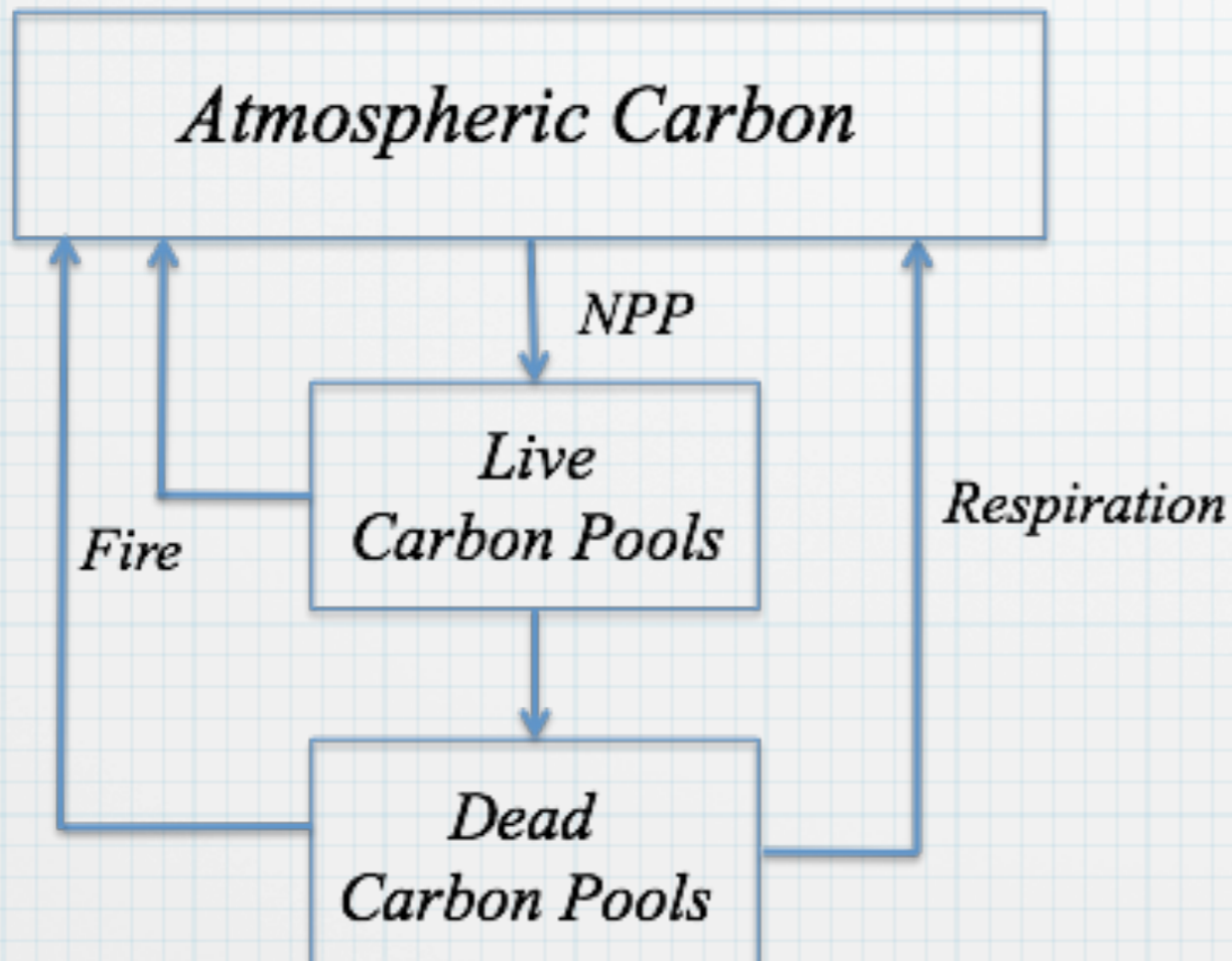
$$\beta A^2 - A(J_P + J_S) + J_P J_S = 0$$



**Chris Field**



**Hal Mooney**



← Atmospheric GCM

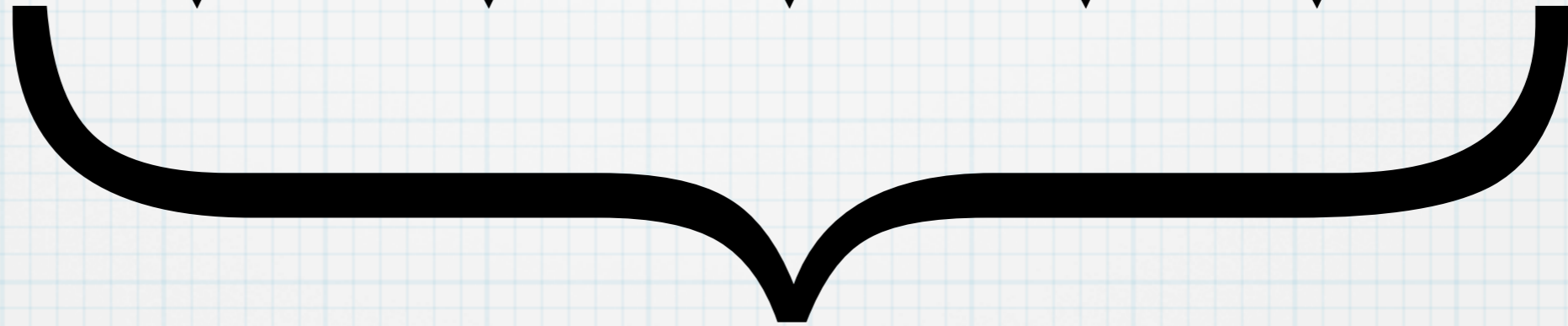
← Land Surface

← Remote Sensing

← Plant Physiology

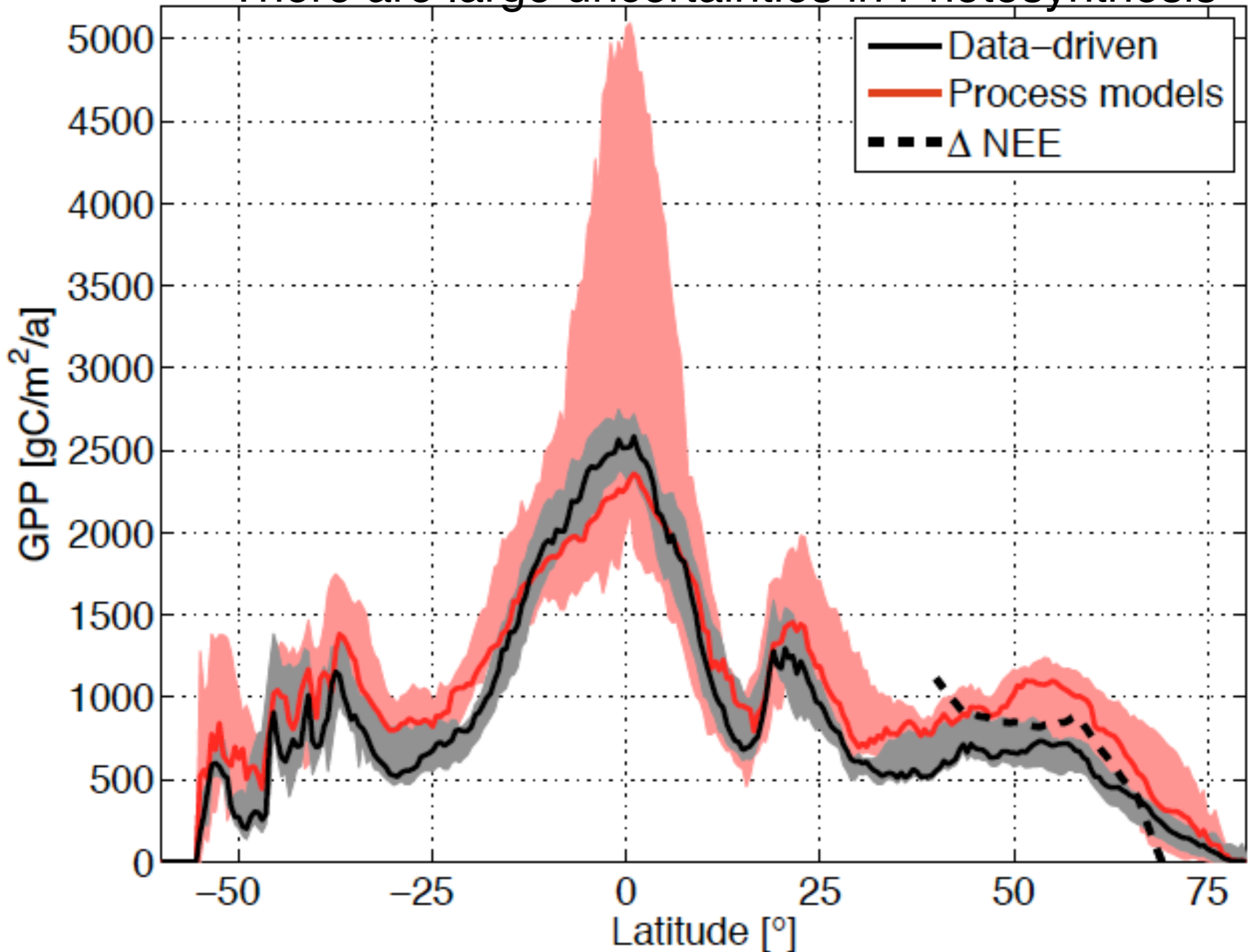
← Ecology

← Carbon Cycle



The “Greening” of the CSU-GCM  
(Sellers - Mooney EOS Team)

# There are large uncertainties in Photosynthesis



# Sellers 1987, Quantitative Remote Sensing

$$\bar{\mu} \frac{dI \downarrow}{dL} + [1 - (1 - \beta)\omega] I \downarrow - \omega\beta I \uparrow$$

$$= \bar{\omega}\mu K(1 - \beta_0)e^{-KL}, \quad (2b)$$

$I \uparrow, I \downarrow$  = upward and downward diffuse radiative fluxes, normalized by the incident flux,

$\mu$  = cosine of the zenith angle of the incident beam,

$K$  = optical depth of direct beam per unit leaf area

$$= G(\mu)/\mu,$$

$G(\mu)$  = relative projected area of leaf elements in direction  $\cos^{-1} \mu$ ,

$\bar{\mu}$  = average inverse diffuse optical depth per unit leaf area

$$= \int_0^1 [\mu' / G(\mu')] d\mu',$$

$\mu'$  = direction of scattered flux,

$\omega$  = scattering coefficient

$$= \alpha + \tau,$$

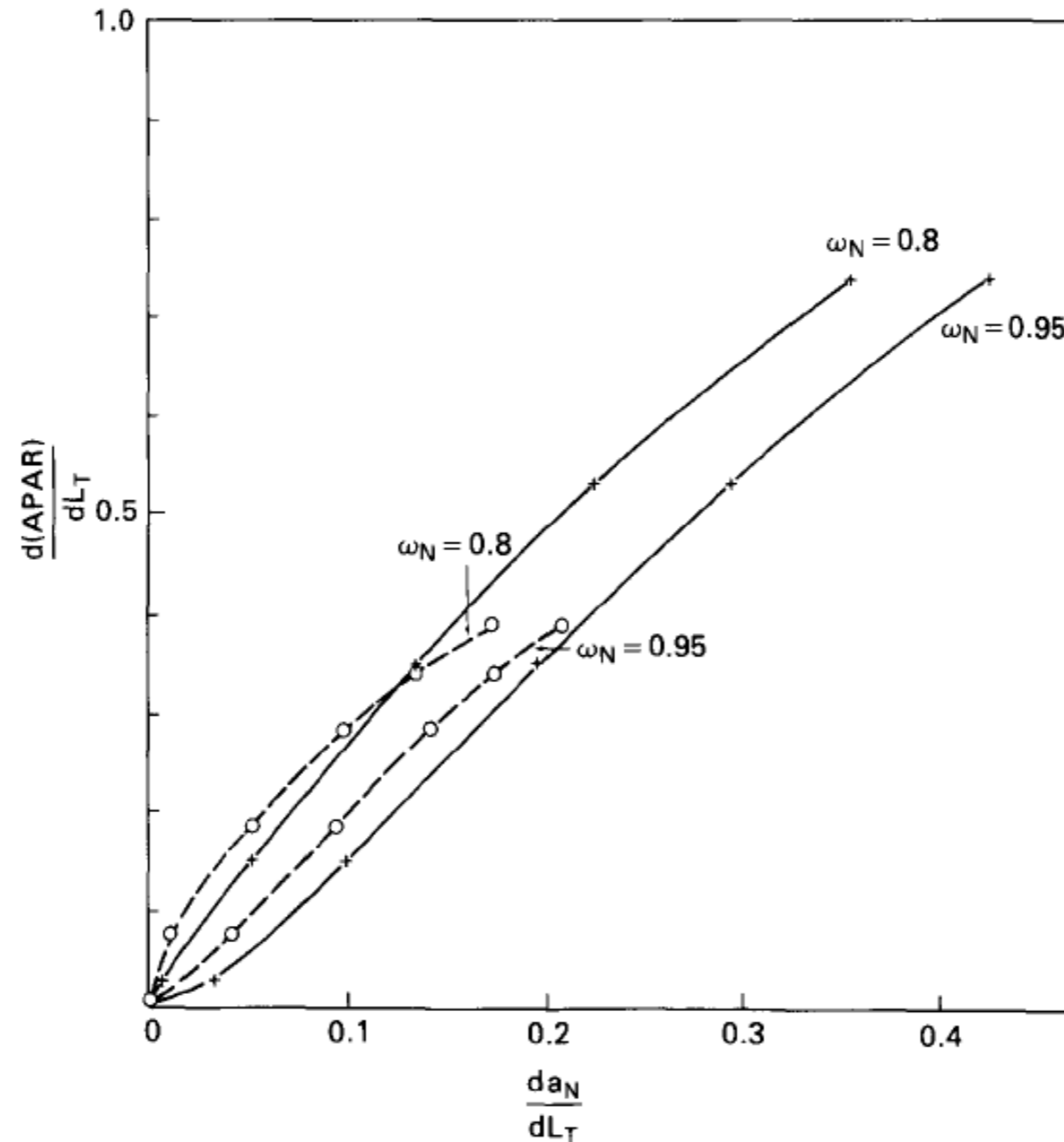
$\alpha$  = leaf element reflectance,

$\tau$  = leaf element transmittance,

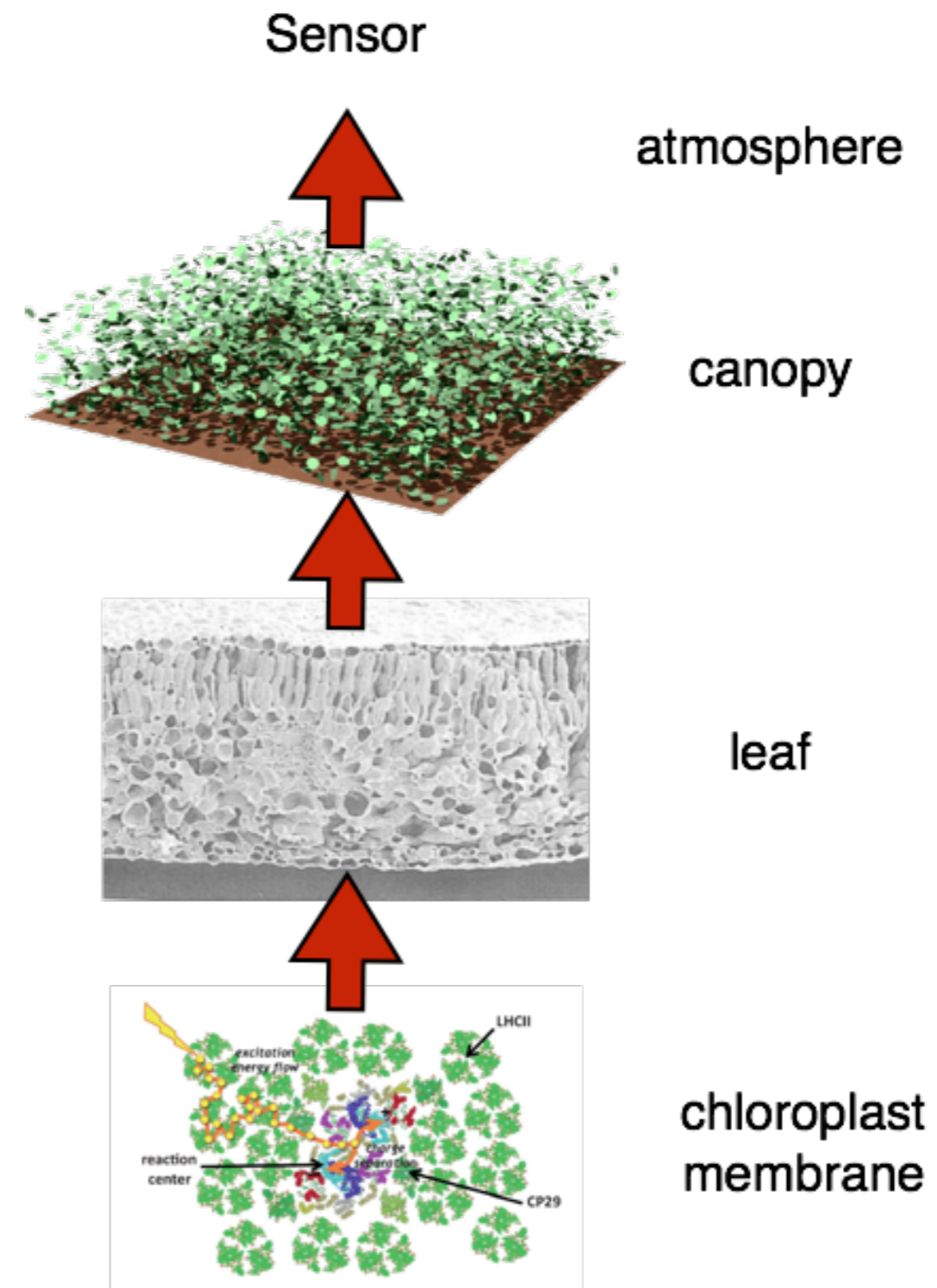
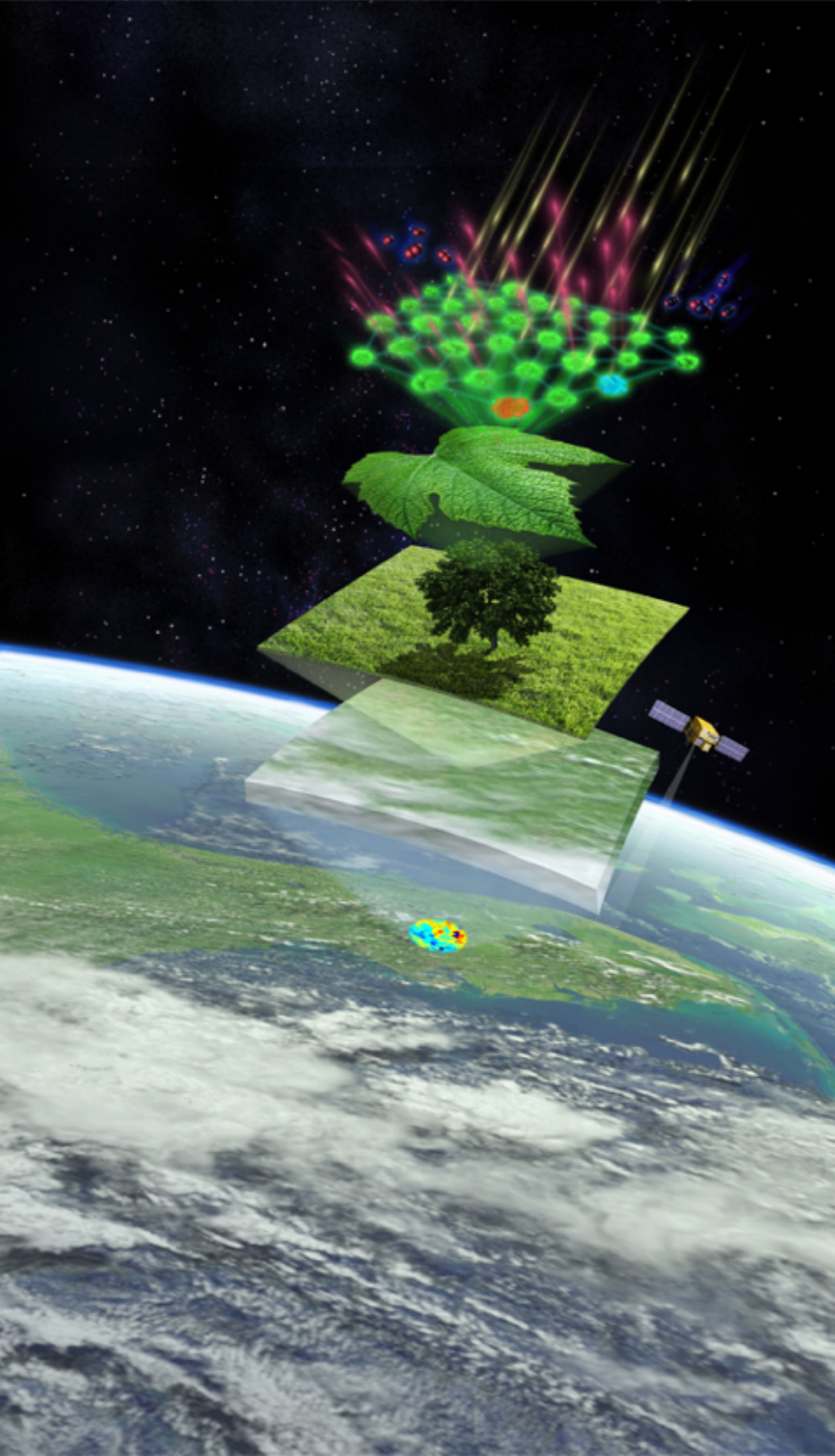
$L$  = cumulative leaf area index,

$\beta, \beta_0$  = upscatter parameters for diffuse and direct beams, respectively.

Sellers showed that NIR reflectance is strongly correlated with APAR



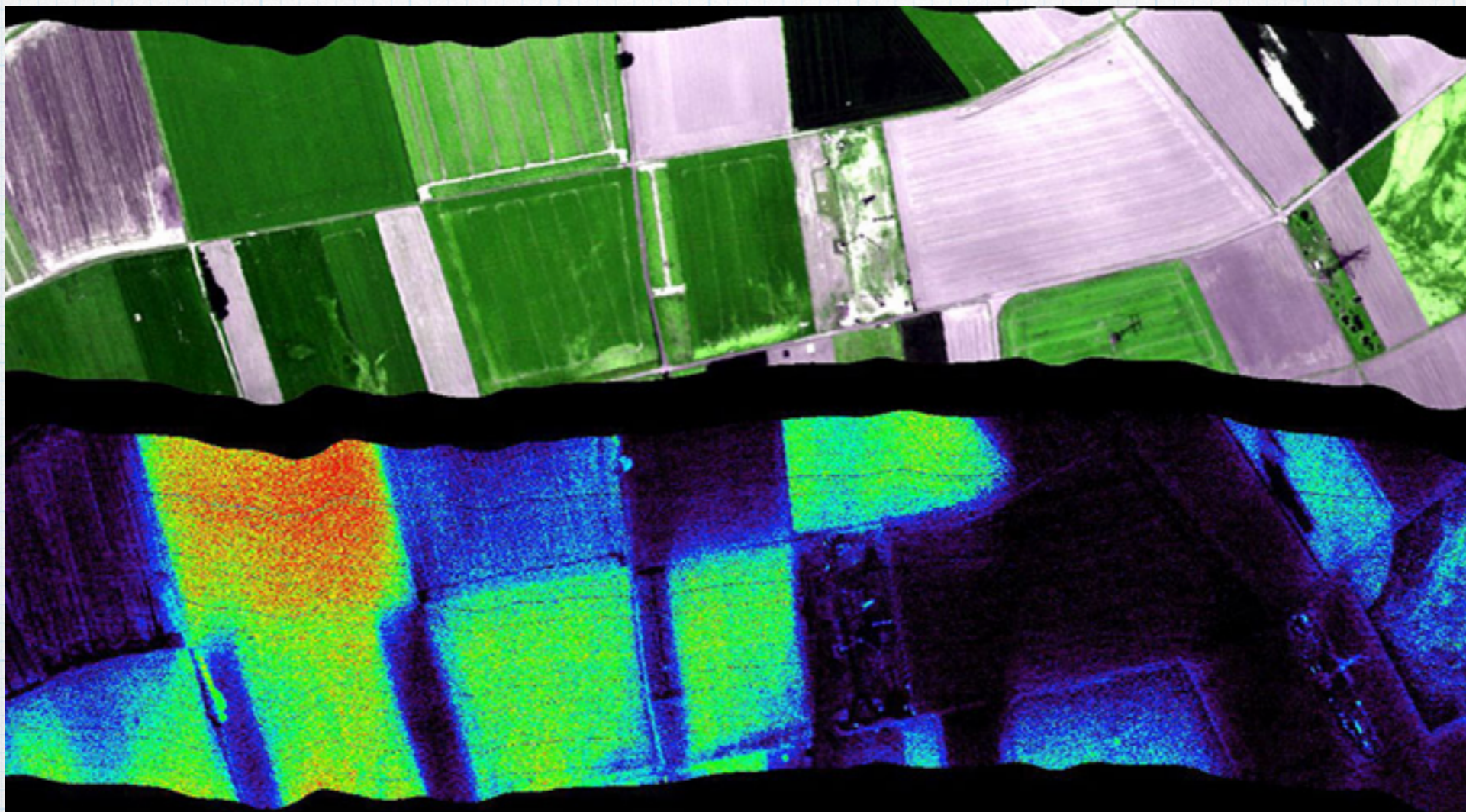
However, NIR reflectance from other materials in mixed scene would contaminate this nice relationship.



Fluorescence arriving at the sensor is emitted in the chloroplast and modified by radiation transport through a hierarchy of scales

# HyPlant (aircraft) Image

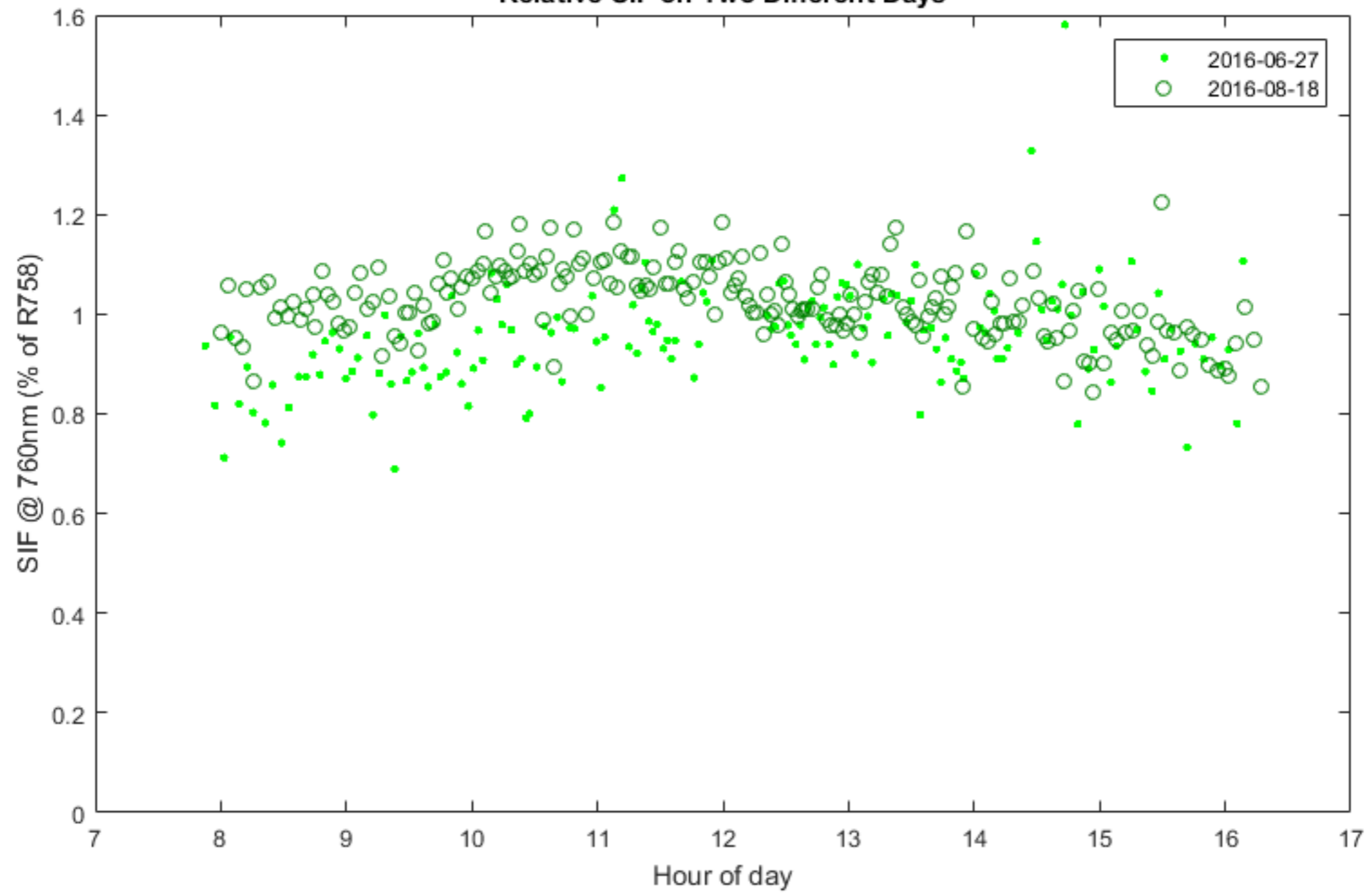
- SIF is specific to vegetation
- SIF varies independently to greenness
- Variation is larger than expected



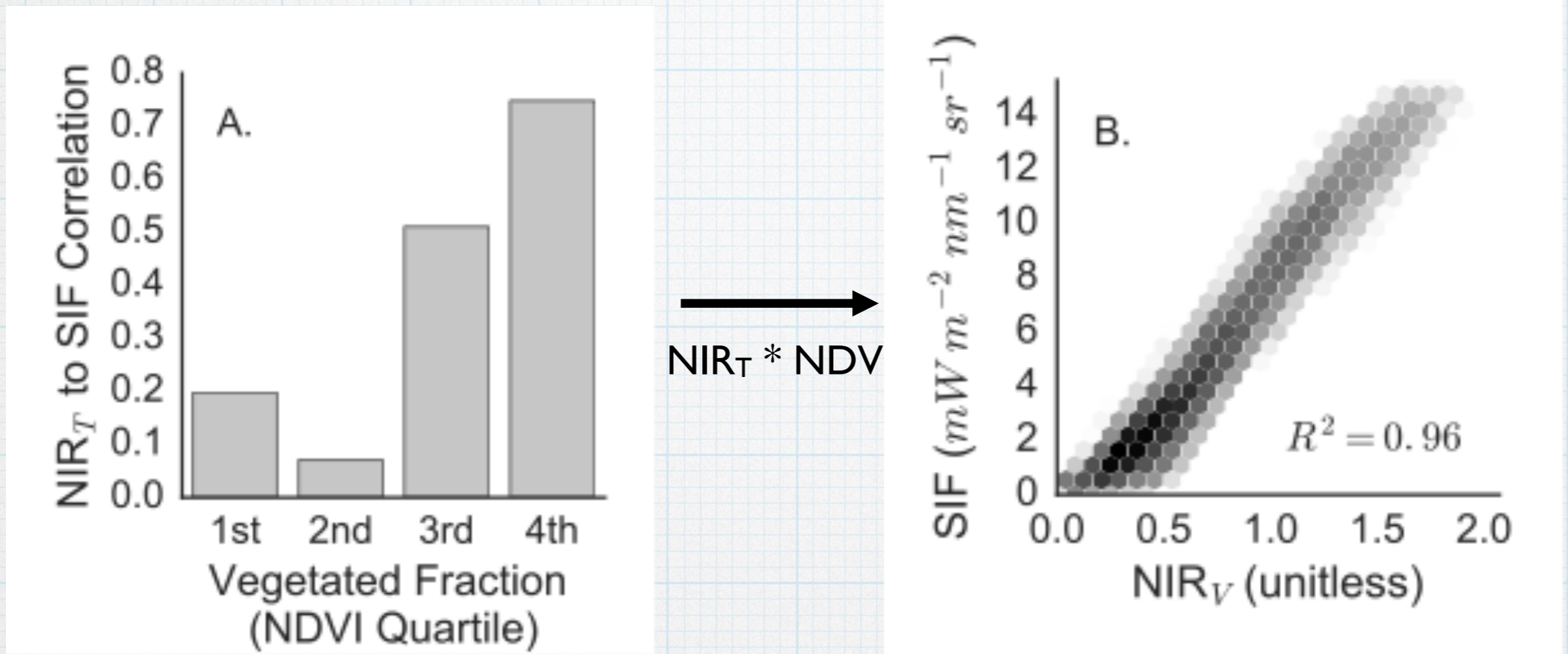




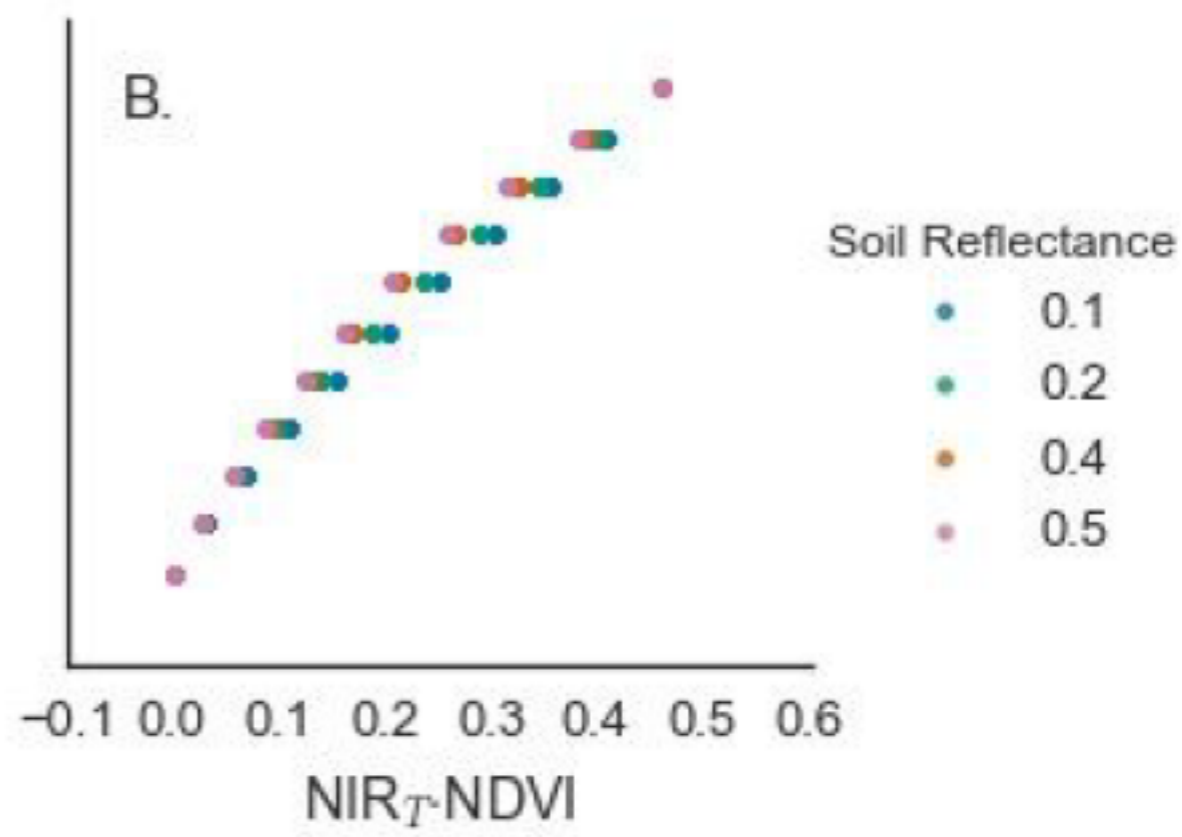
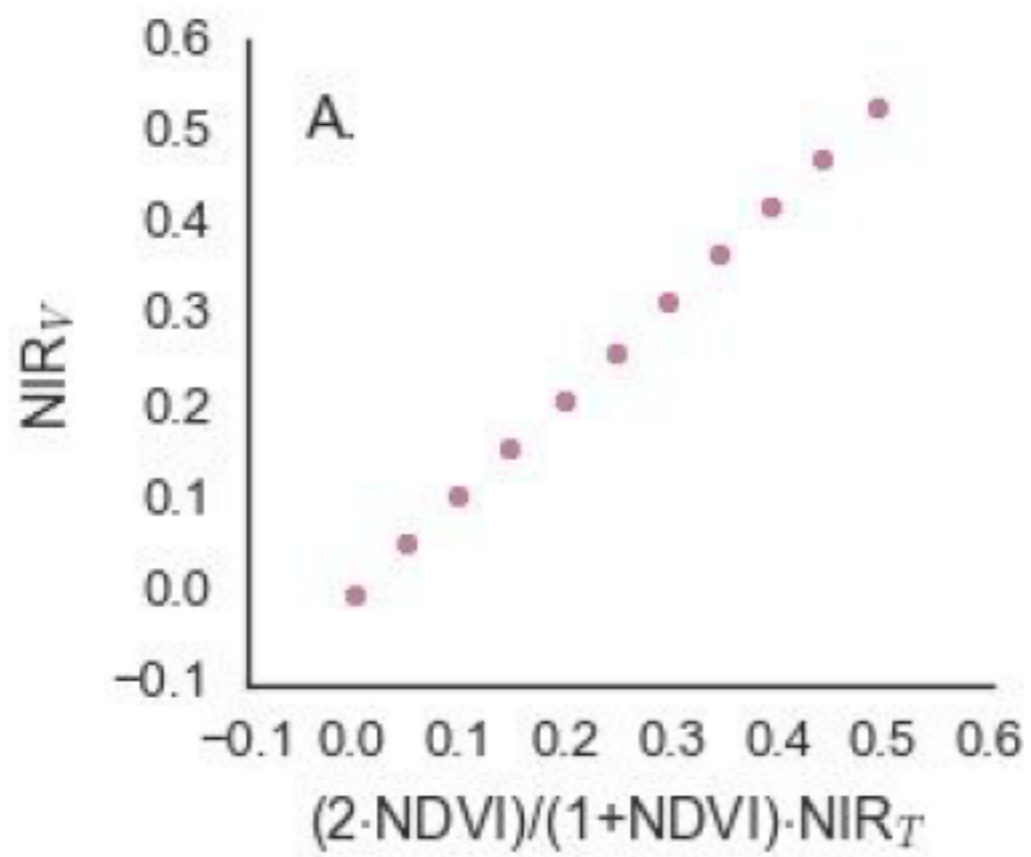
Relative SIF on Two Different Days



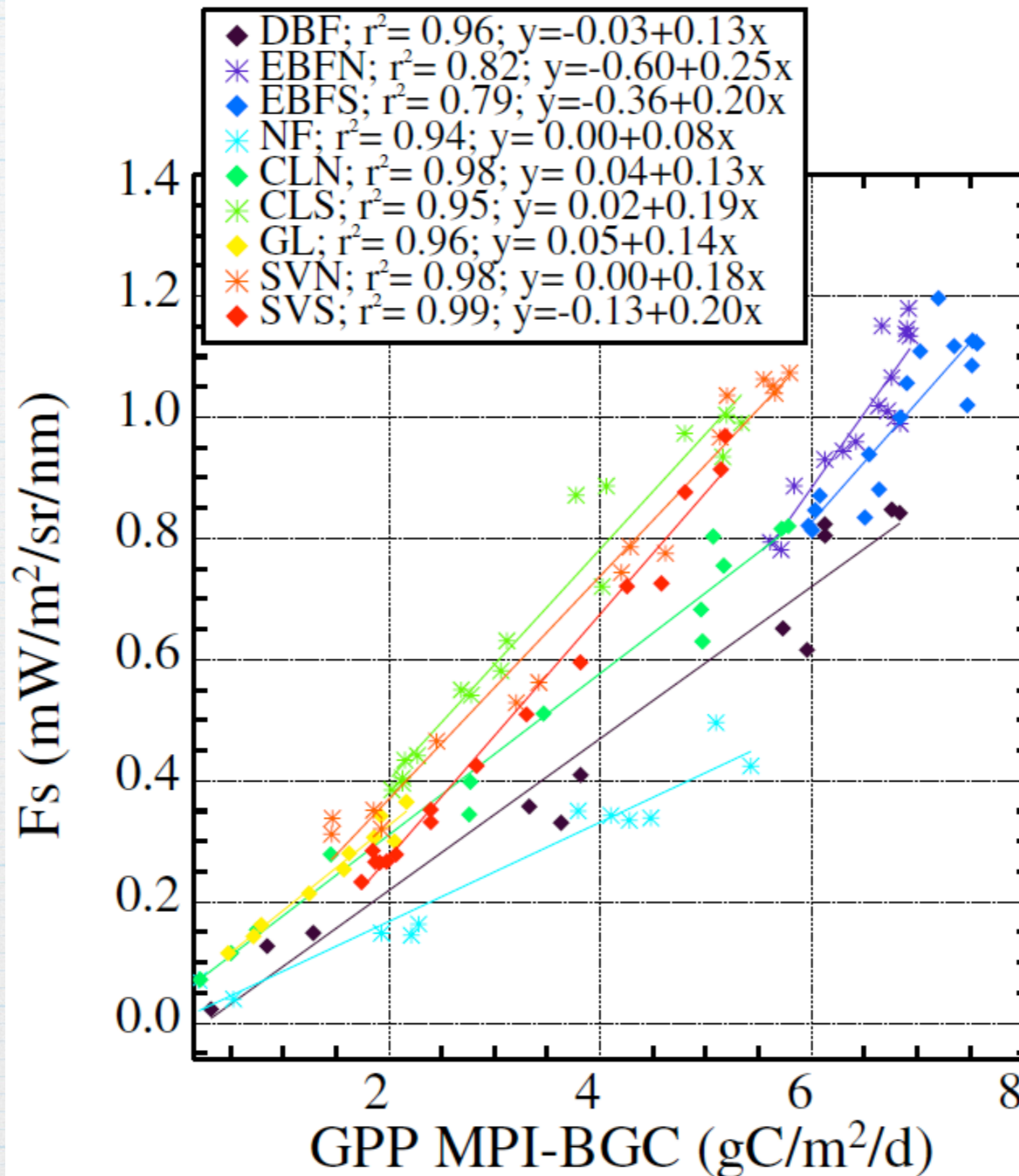
# SIF Correlates with NIR Reflectance



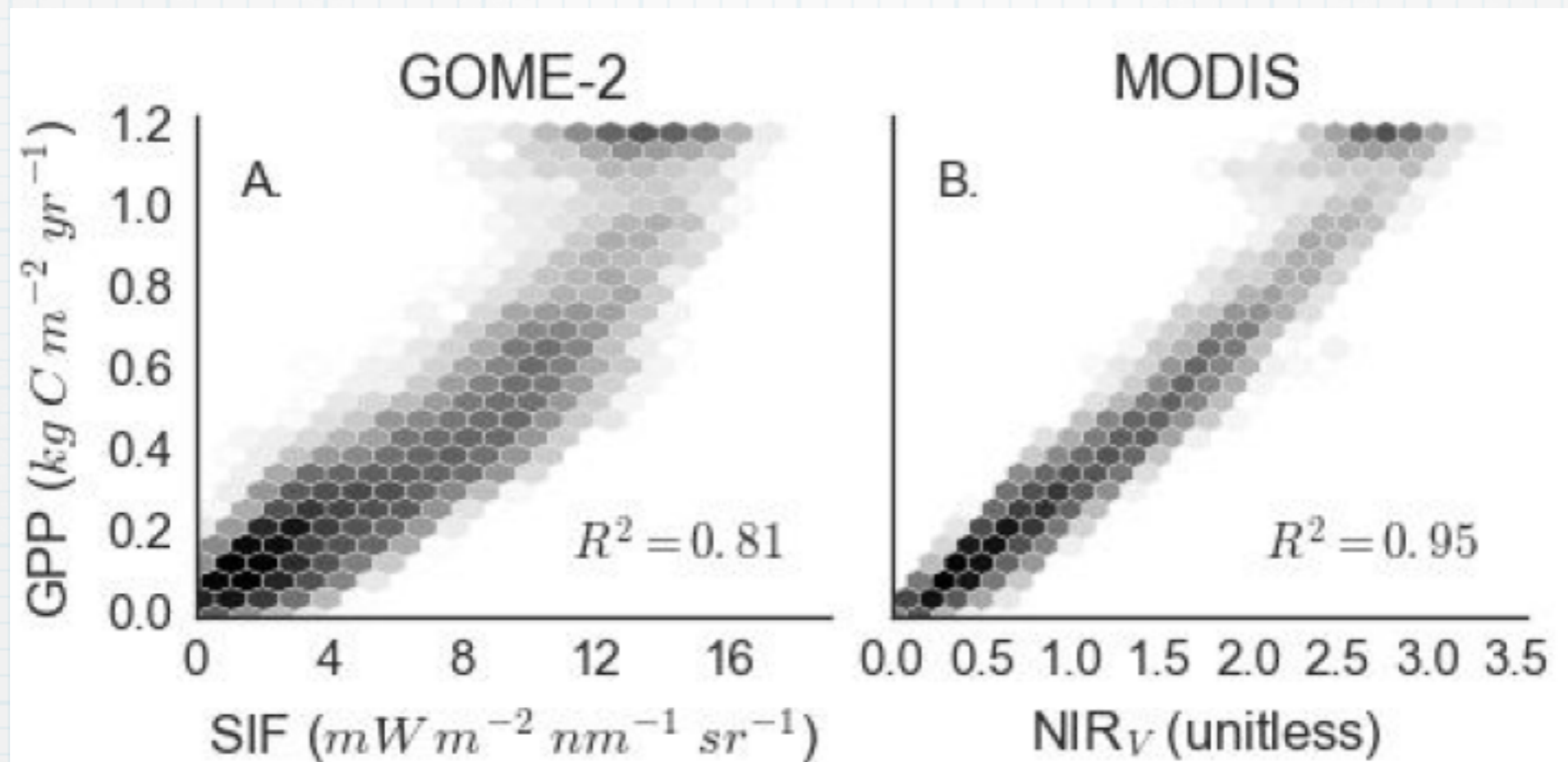
We have come up with a new way to use MODIS data to obtain a metric, the NIR<sub>V</sub>, which we consider to be the NIR reflection from the vegetation of a mixed scene. It is approximated as  $NIR_V = NIR_T * NDVI$  and it correlates remarkably well with SIF and GPP.

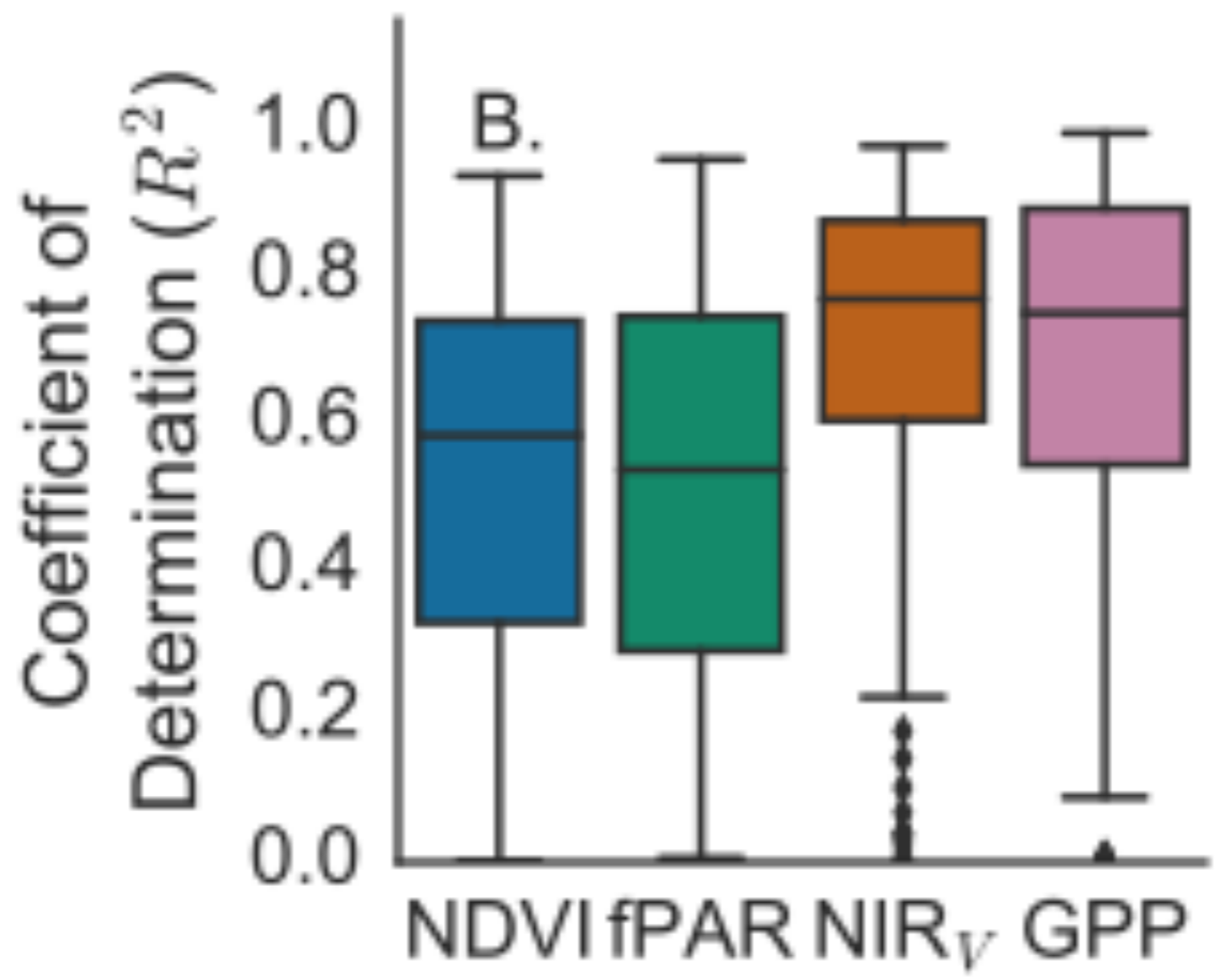


## SIF Correlates Strongly with (MPI) GPP



# MPI-GPP at $0.5^\circ$ vs SIF (GOME-2) or NIR<sub>v</sub> (MODIS)



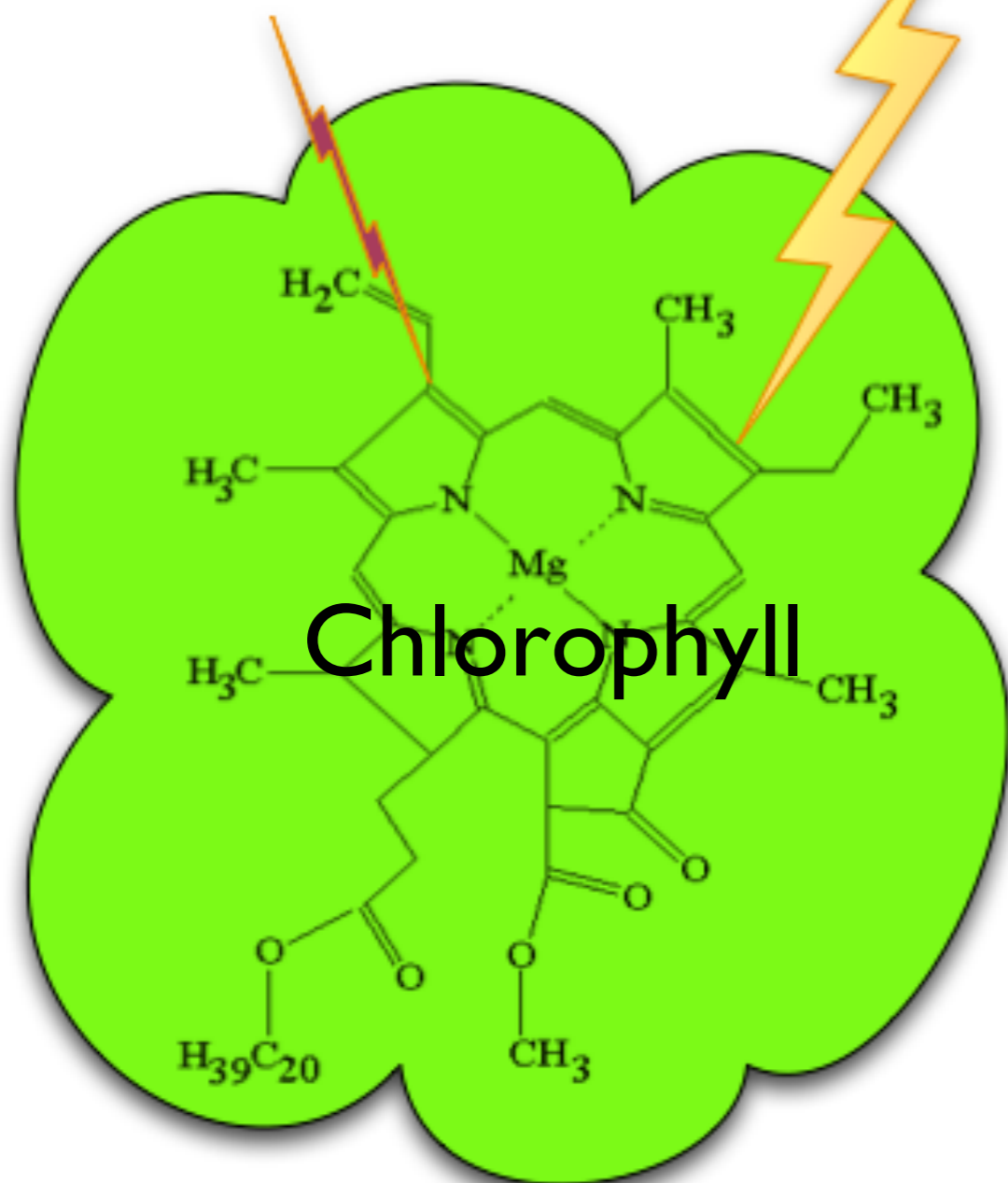


We have come up with a new way to use MODIS data to obtain a metric, the NIR<sub>v</sub>, which we consider to be the NIR reflection from the vegetation of a mixed scene. It is approximated as  $NIR_V = NIR_T * NDVI$  and it correlates remarkably well with SIF and GPP. This appears to be a subtle feature of canopy structure.

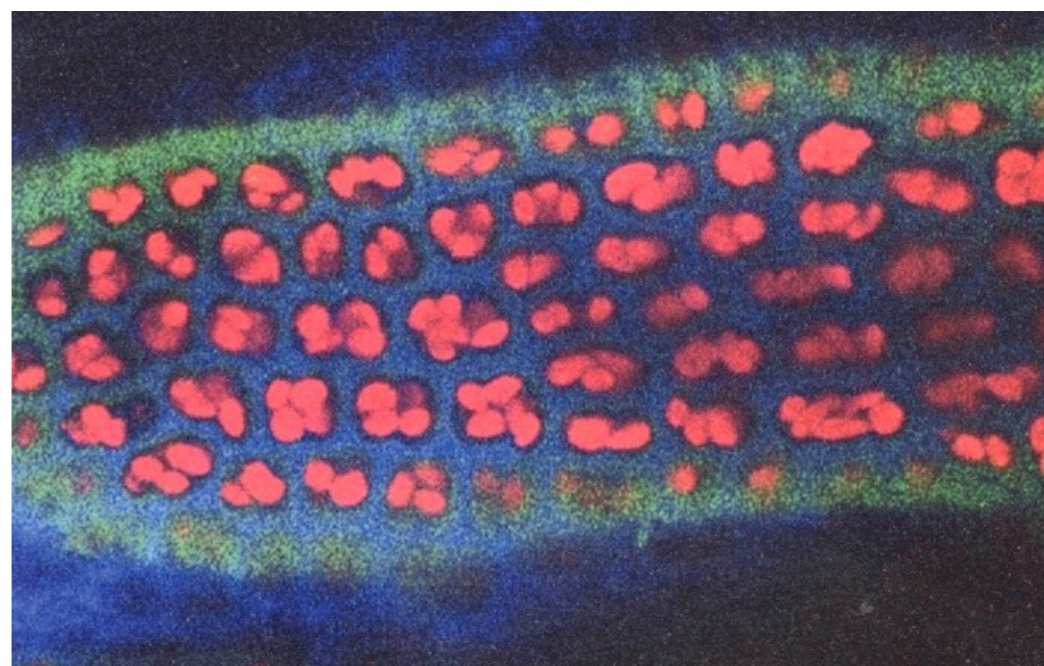
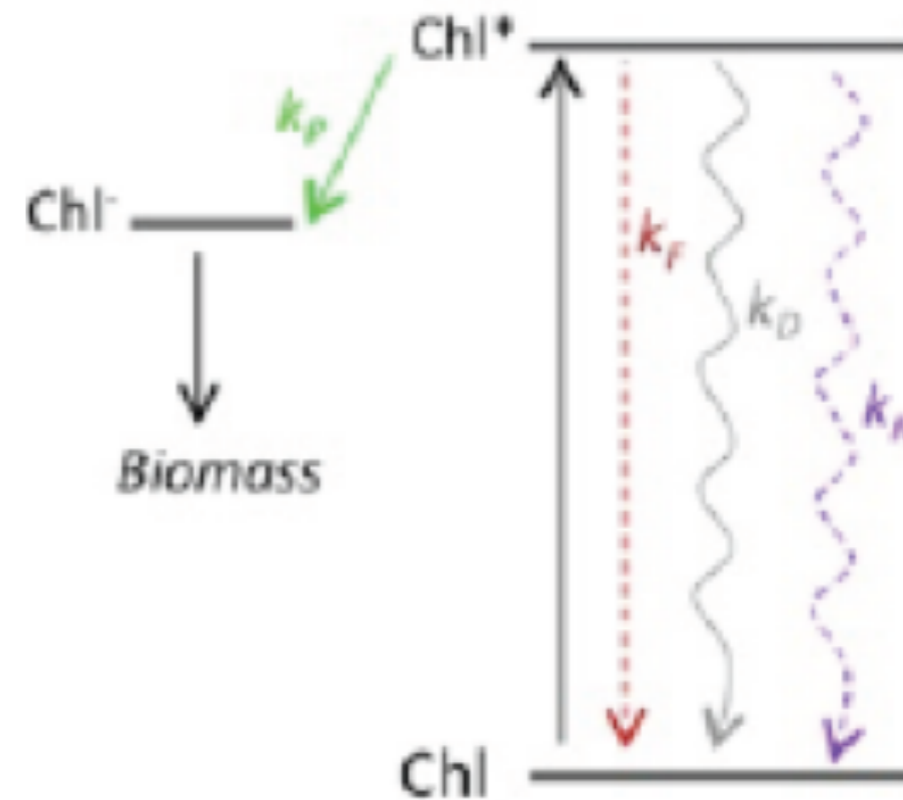


Fluorescence  
photon

Solar  
photon

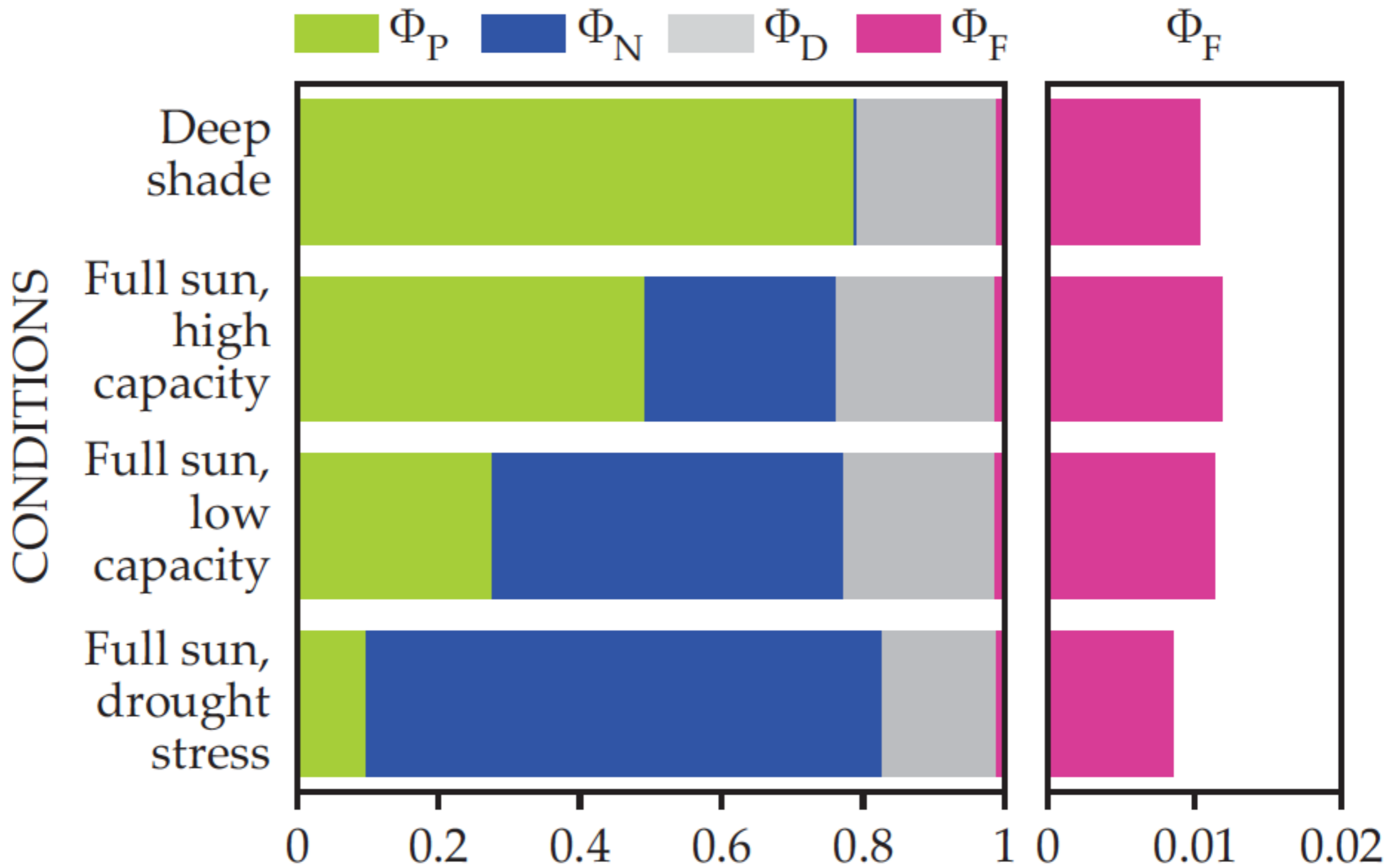


Chlorophyll



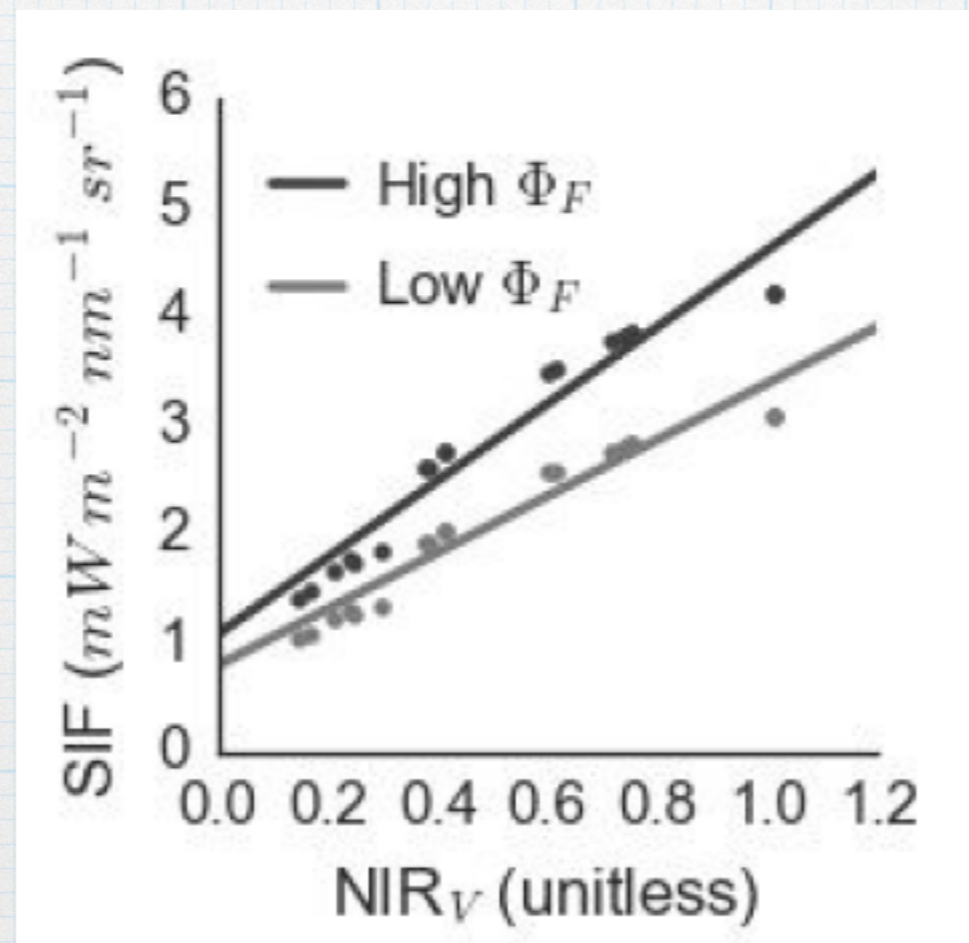
$$\Phi_F = K_F / K_T$$

Where  $K_F$  is a constant



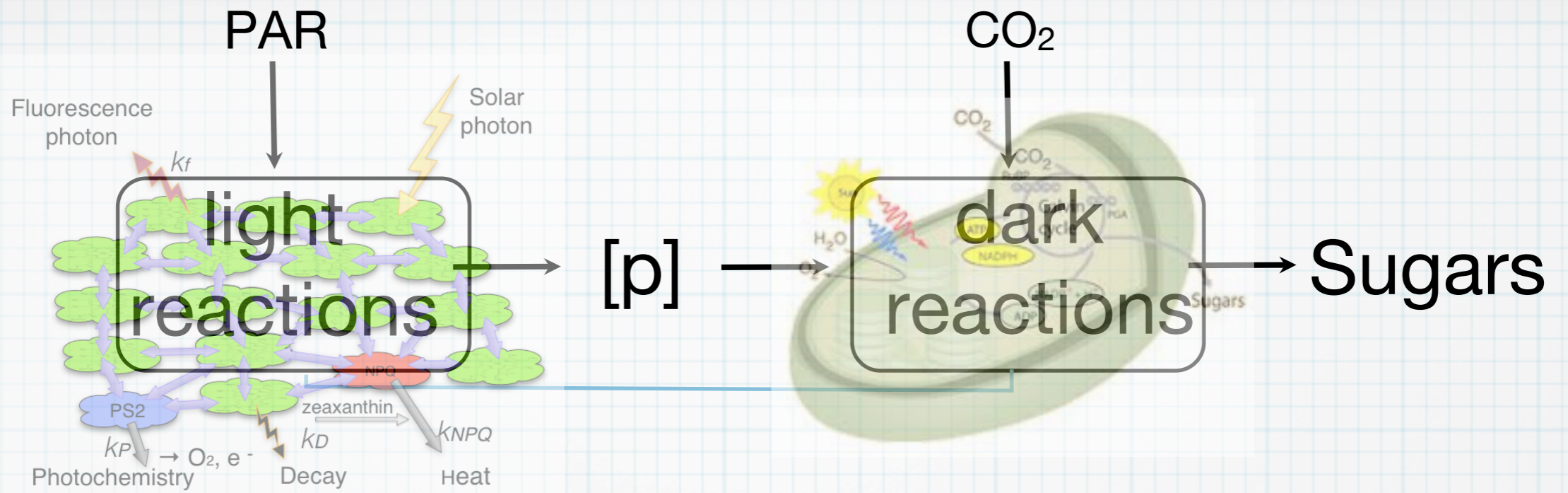
Schlau-Cohen, G. S., & Berry, J. (2015). Photosynthetic fluorescence, from molecule to planet. *Physics Today*, 68(9), 66–67. <http://doi.org/10.1063/PT.3.2924>

- ▼ Modeling studies with SCOPE indicate that  $\text{NIR}_V$  permits us to separate control of SIF into 2 components.
  - Physiological control of the fluorescence yield
  - Leaf display and light interception
  - Remarkably BOTH seem to be correlated with GPP

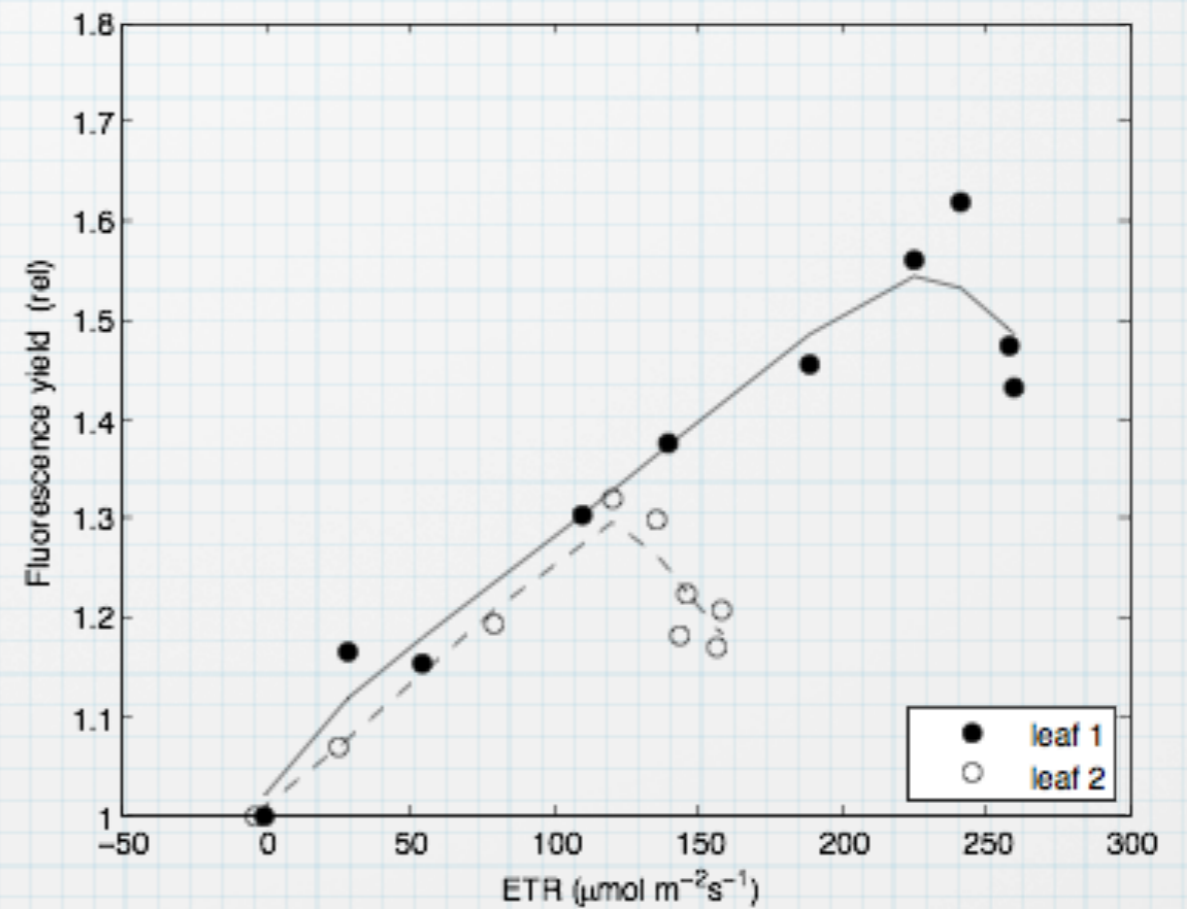
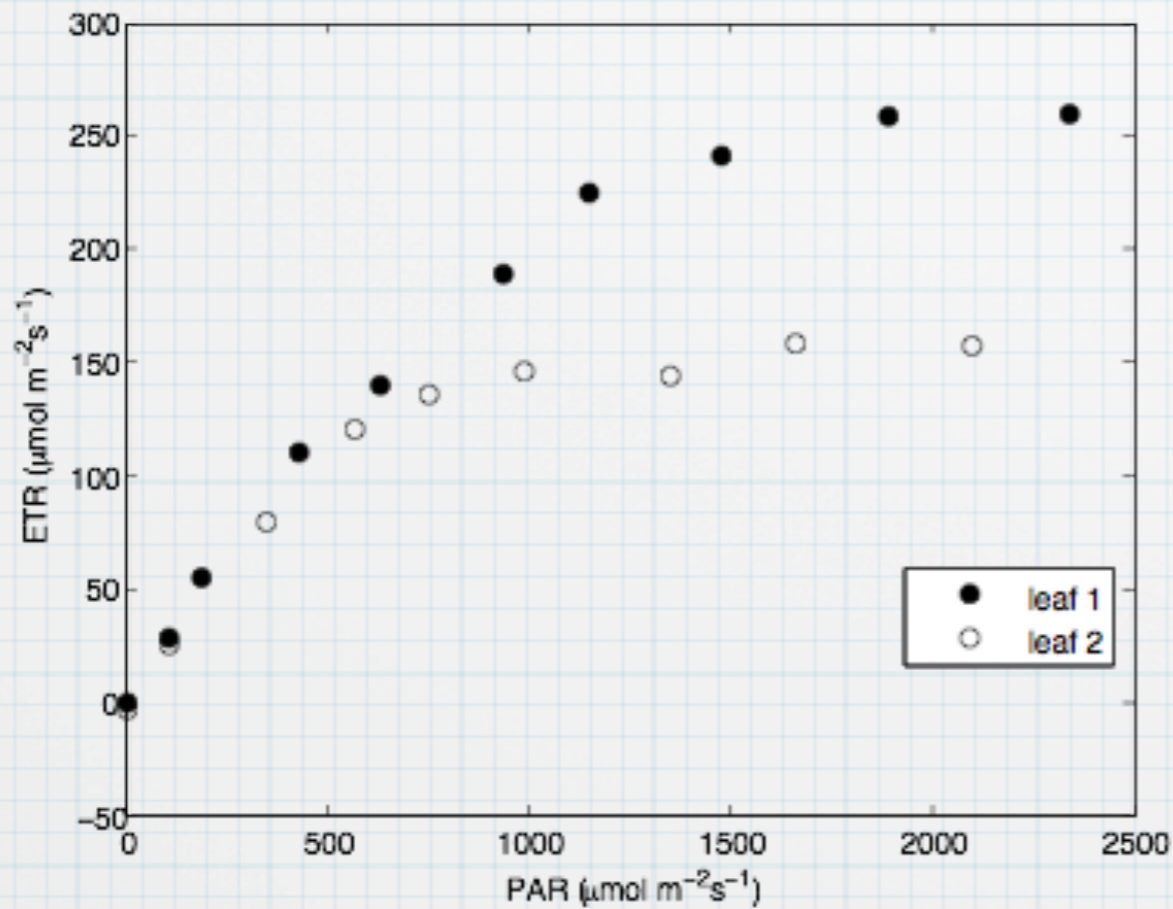


## Final Thoughts:

- SIF is turning out to be surprisingly useful:
  - Seems to be proportional to GPP;
  - Indicates drought;
  - Indicates beginning and end of growing season.
- It is also a hot topic in fundamental research on photosynthesis.
- Fluorescence is a product of photosynthesis which can be modeled and measured over the globe. If we model SIF correctly does it mean that we have modeled GPP correctly?
- We have come up with a new way to use MODIS data to obtain a metric, the NIR<sub>v</sub>, which we consider to be the NIR reflection from the vegetation of a mixed scene. It is approximated as  $NIR_v = NIR_t * NDVI$  and it correlates remarkably well with SIF and GPP.

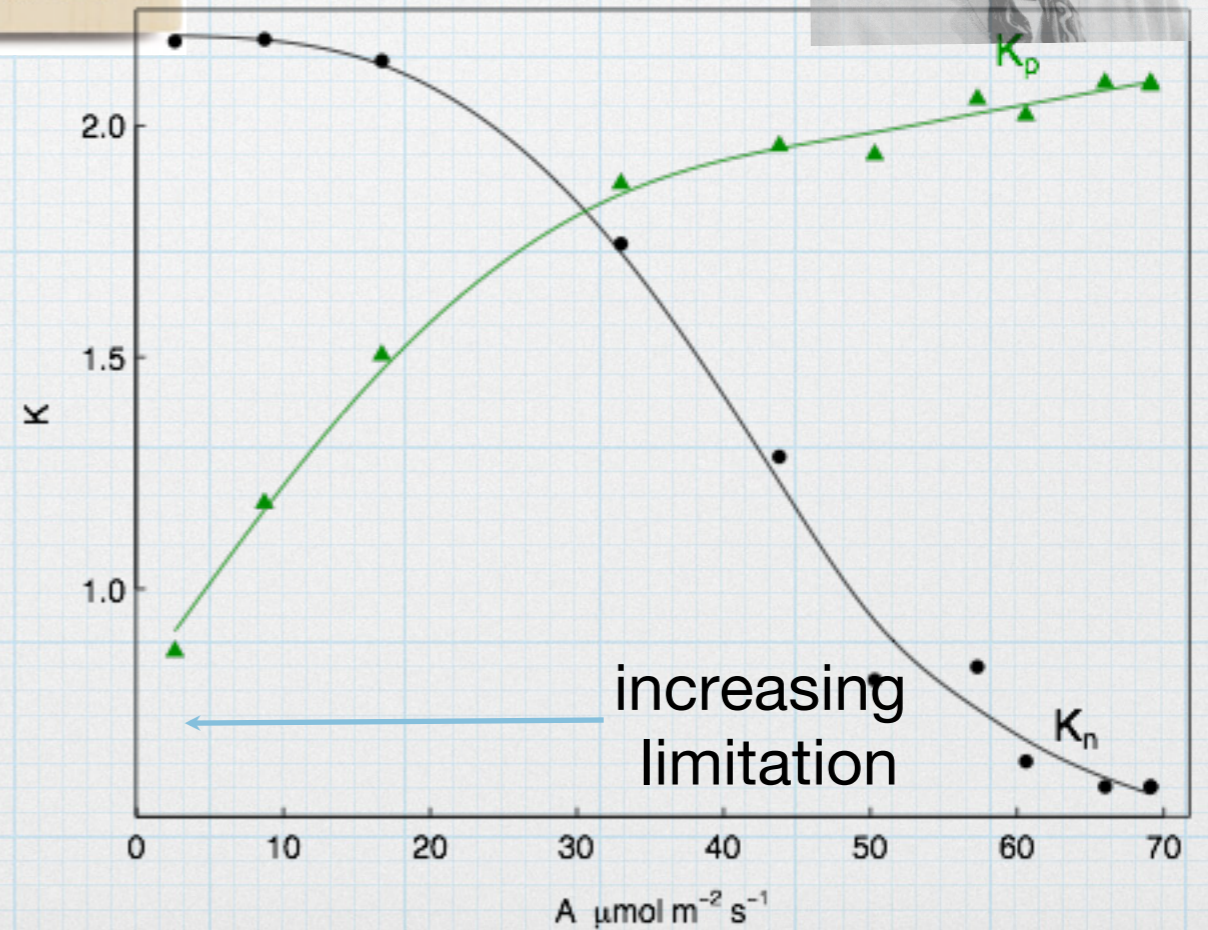
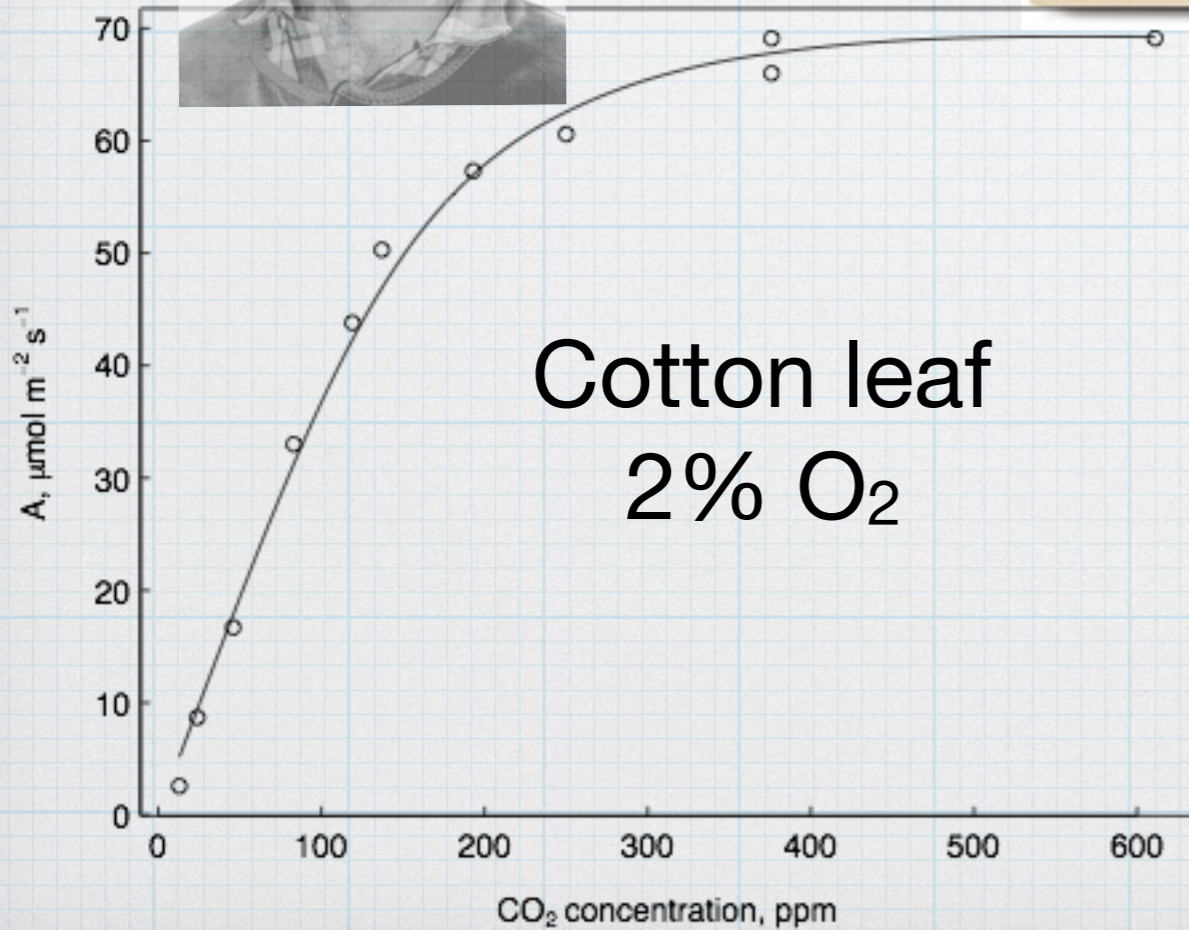
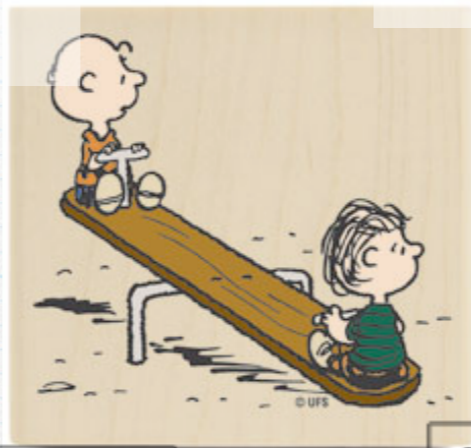
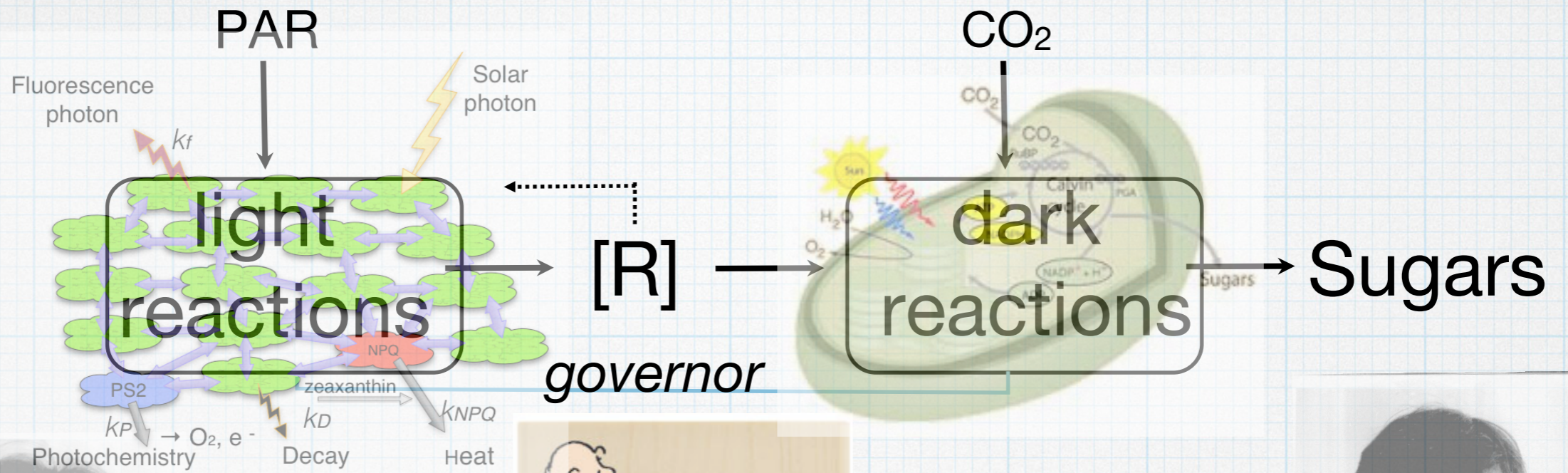


## Fluorescence vs. the electron transport rate (ETR)

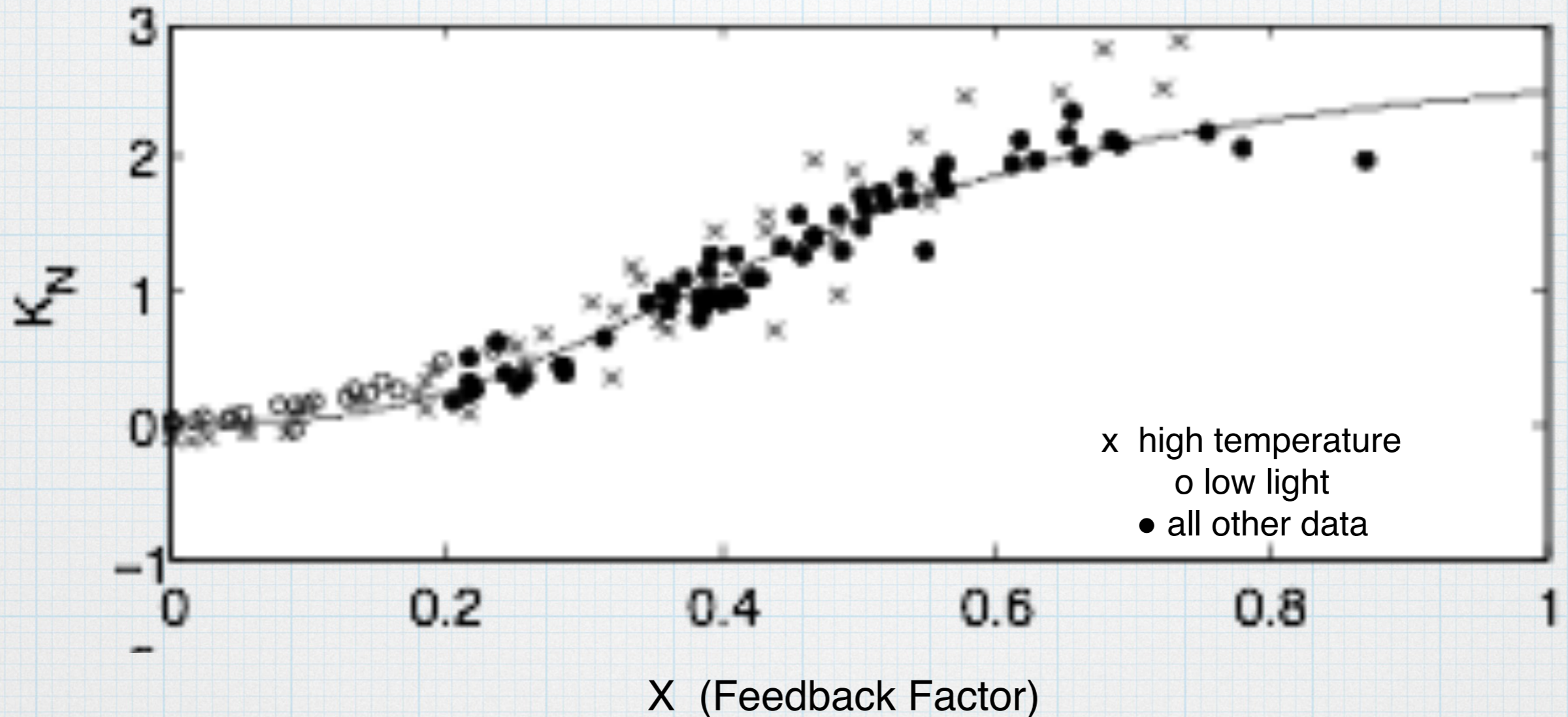


# Summary

- Studies of the mechanisms controlling fluorescence provide a good basis for predicting the yield of solar induced fluorescence (SIF).
- Feedback mechanisms appear to keep fluorescence yield fairly constant.
- Stress (cold, drought, heat) generally tends to reduce fluorescence.
- Observations of SIF generally show larger changes than predicted based on physiological parameterizations. Seems like structure also plays a role



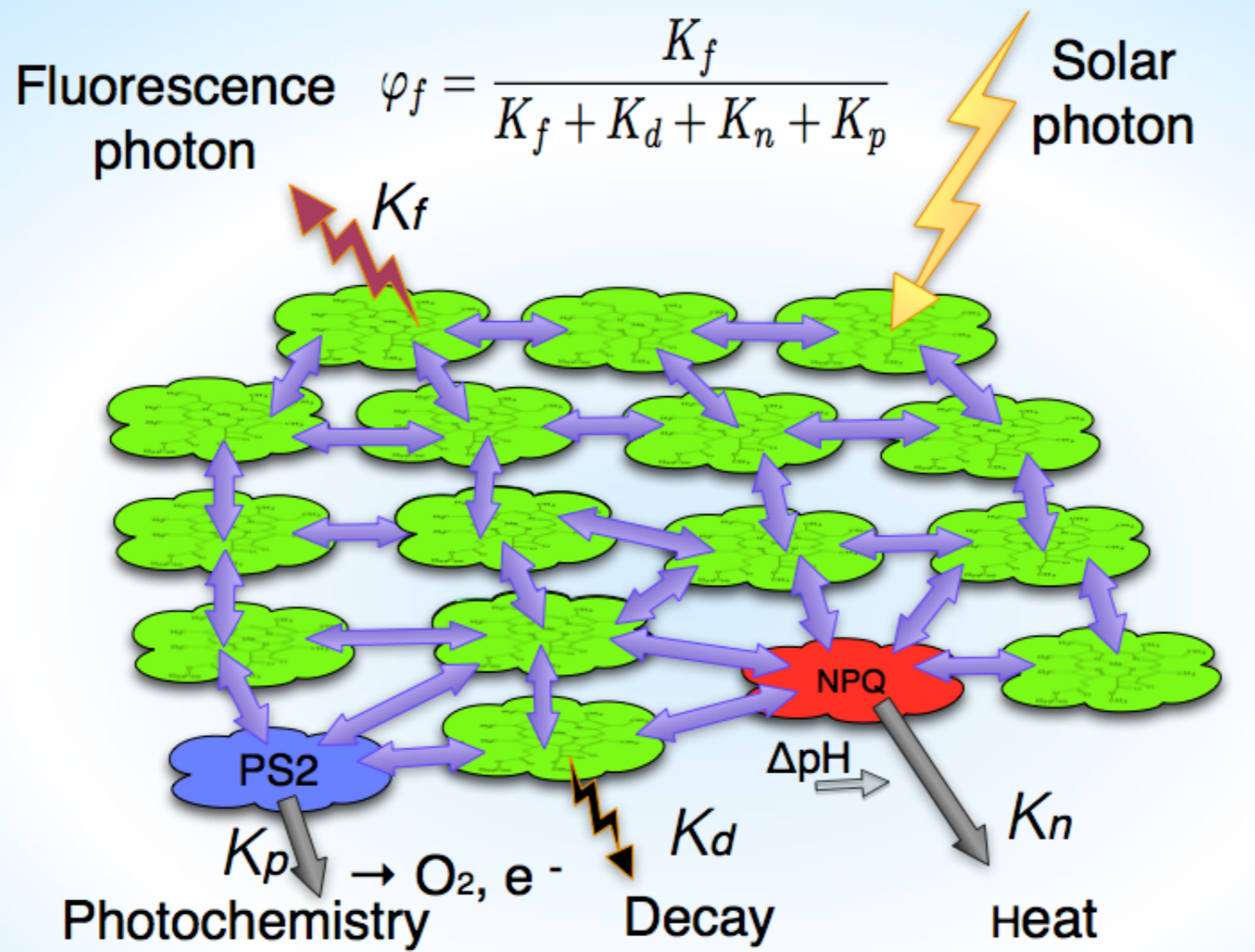
## Finding and empirical expression for $K_N$



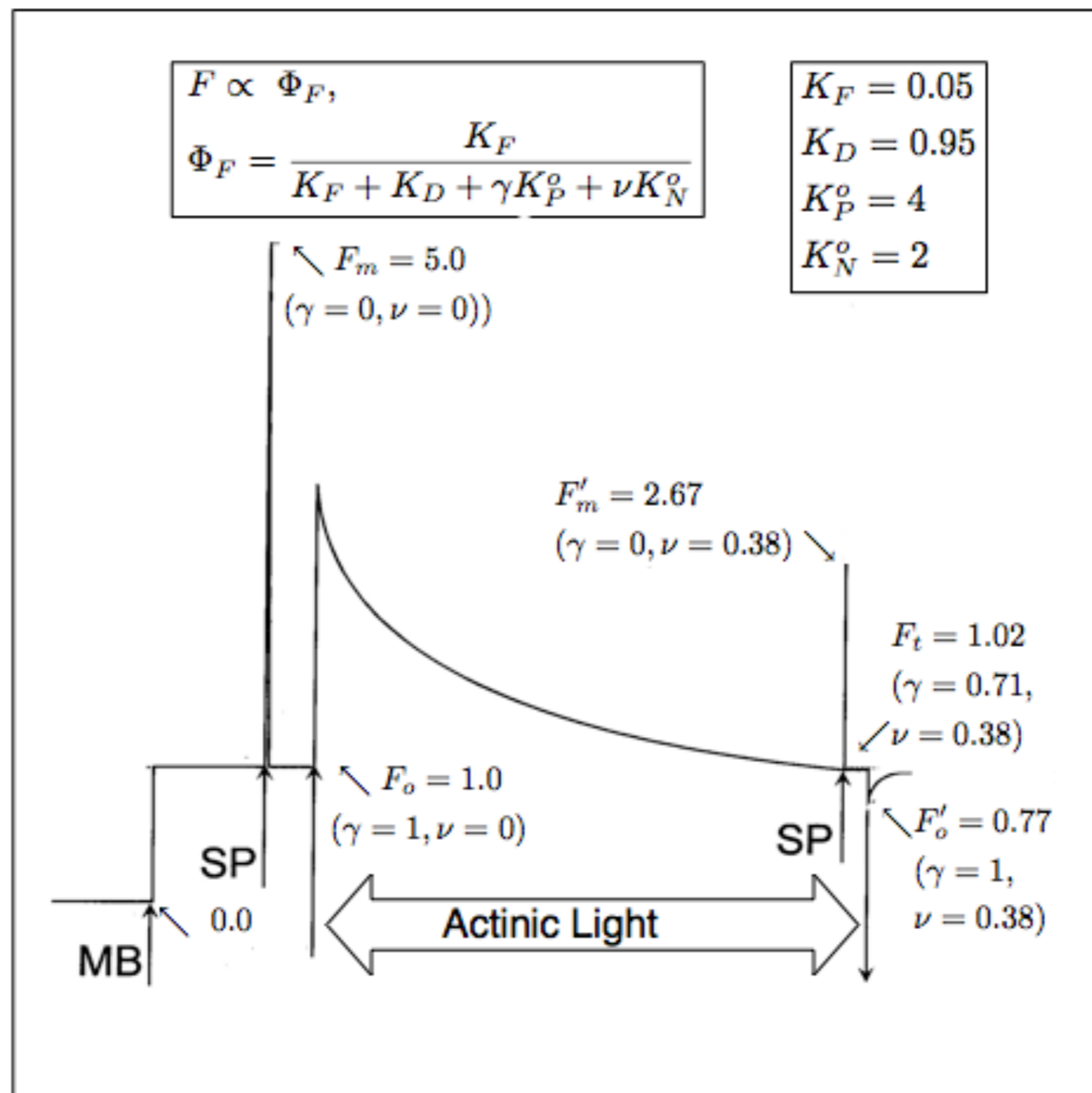
$$K_N = \nu K_N^o \quad \text{with} \quad \nu = \frac{(1 + \beta)x^\alpha}{\beta + x^\alpha}$$

Tol, C., Berry, J. A., Campbell, P. K. E., Campbell, P., & Rascher, U. (2014). Models of fluorescence and photosynthesis for interpreting measurements of solar-induced chlorophyll fluorescence. *Journal of Geophysical Research-Biogeosciences*, 119(12), 2312–2327. <http://doi.org/10.1002/2014JG002713>



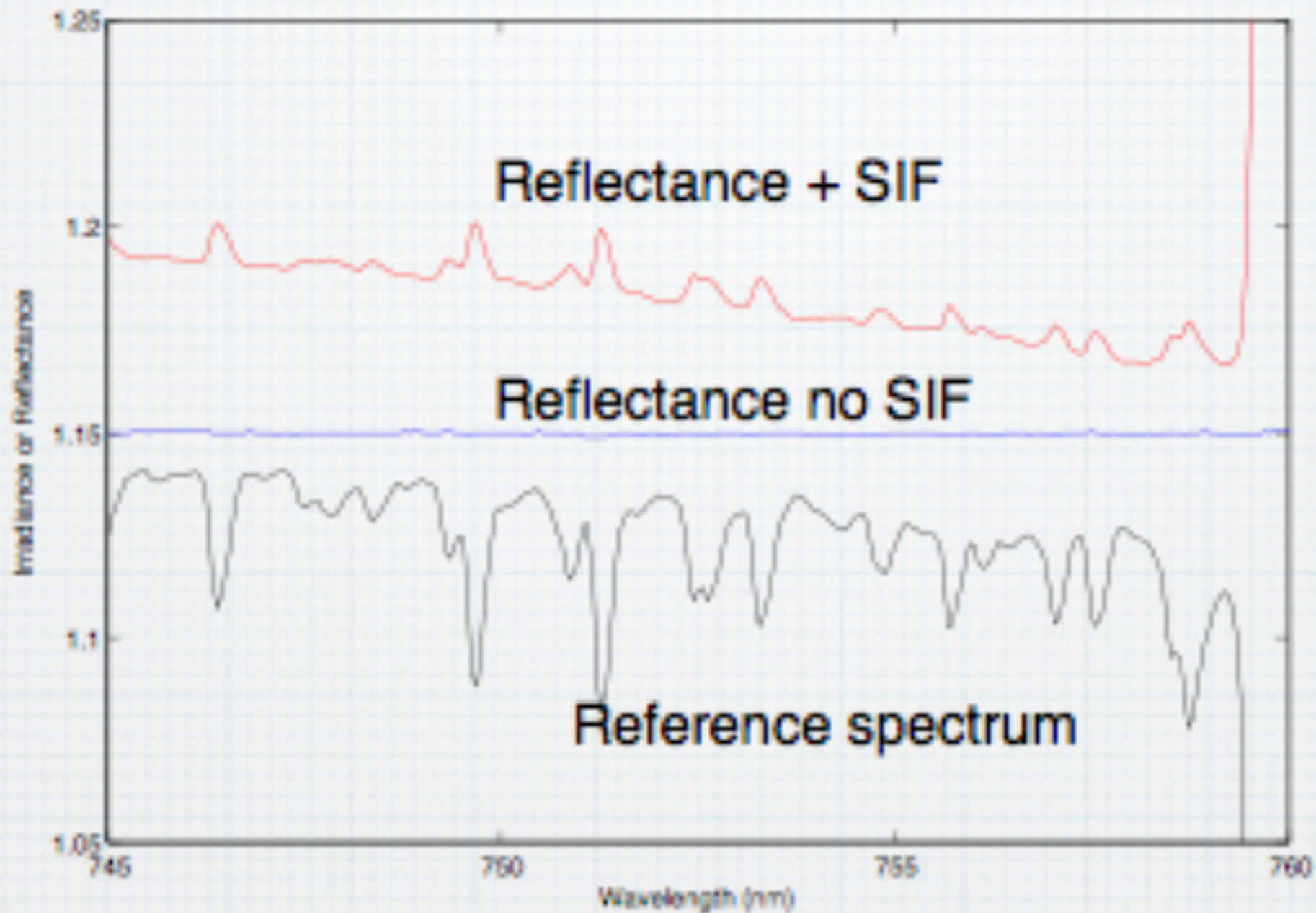


## Relating PAM levels to absolute yields



Tol, C., Berry, J. A., Campbell, P. K. E., Campbell, P., & Rascher, U. (2014). Models of fluorescence and photosynthesis for interpreting measurements of solar-induced chlorophyll fluorescence. *Journal of Geophysical Research-Biogeosciences*, 119(12), 2312–2327. <http://doi.org/10.1002/2014JG002713>

## Digging out Solar Induced Fluorescence (SIF)



Schlau-Cohen, G. S., & Berry, J. (2015). Photosynthetic fluorescence, from molecule to planet. *Physics Today*, 68(9), 66–67. <http://doi.org/10.1063/PT.3.2924>

# Comparison of SIF Capable Satellites

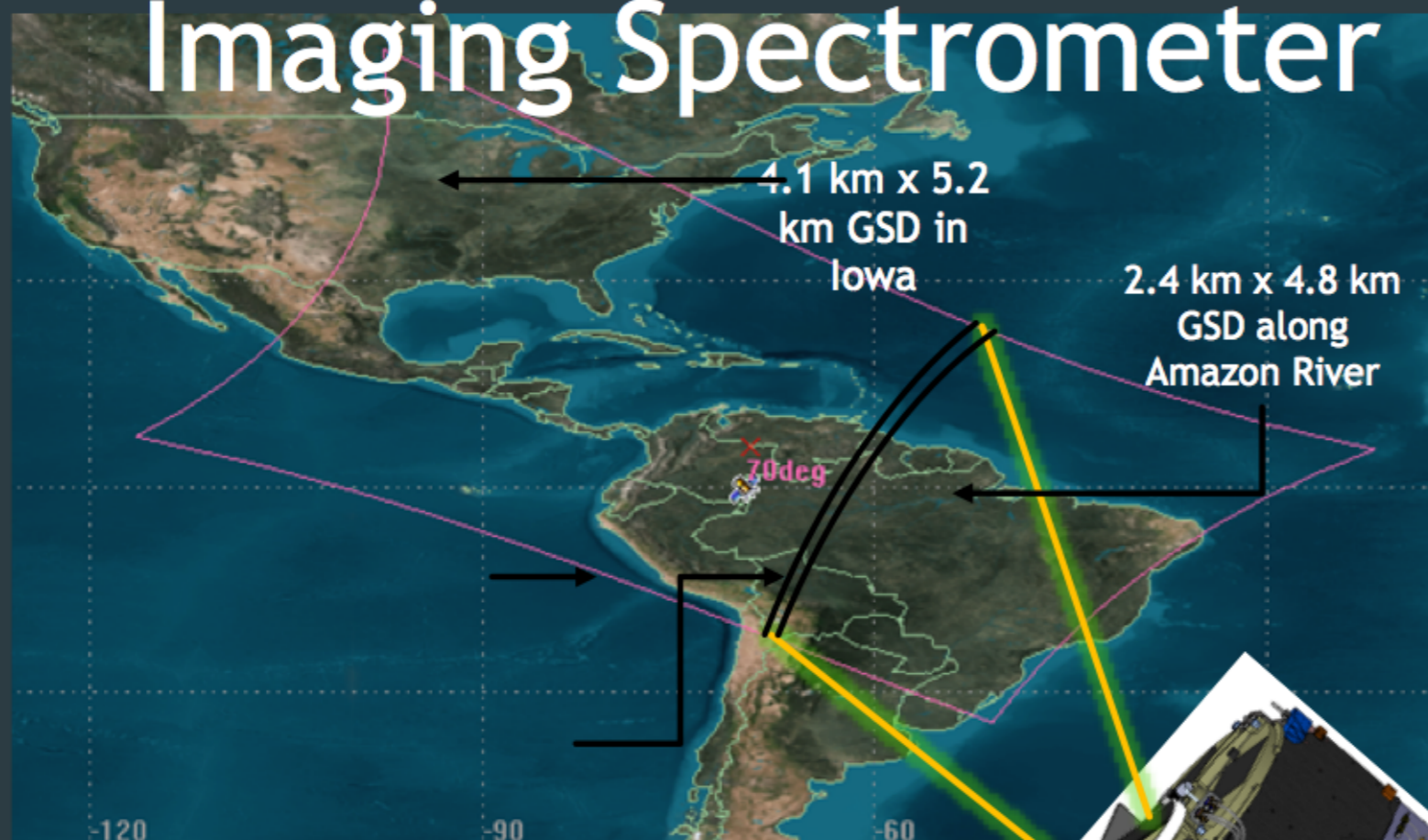
Instrument	Status*	Coverage	Footprint (km)	Revisit time	Spectral Range (nm)	FWHM (nm)	SNR	SIF pixel Quality***
FOLIAGE	F	N/S Amer.	☉ 2.4x4.8	● 2-3 hrs	● 490-790	☉ 0.30	●	●
FLEX <sup>1</sup>	F	56S-75N	● 0.3x0.3	● 19 d	● 500-780	☉ 0.3-2	☉	?
TROPOMI <sup>2</sup>	S	Global	○ 7x7	☉ 1 d	☉ 660-800	● 0.5	●	☉
GOME-2 <sup>3</sup>	O 2007-	Global	● 40x80	☉ 1.5 d	● 270-800	● 0.5	●	☉
TEMPO <sup>4</sup>	S	CONUS	☉ 4x5	● 1 hr	☉ 520-740	● 0.6	☉	☉
OCO-2 <sup>5</sup>	O 2014-	Global**	☉ 2x2	● 16 d	● 755-775	● 0.05	○	☉
GOSAT <sup>6</sup>	O 2009-	Global**	○ 10x10	○ 3 d	● 755-775	● 0.03	●	☉
Sentinel 4 <sup>7</sup>	S	Europe	○ 8x8	● 1 hr	● 755-775	☉ 0.12	○	○
GCPM <sup>8</sup>	F	US	☉ 4x4	● 1 hr	● 756-760	☉ 0.10	☉	○
CarbonSat <sup>9</sup>	F	Global	☉ 2x2	☉ 5 d	☉ 747-773	☉ 0.10	●	☉

●: Excellent; ☉: Very Good; ○: Good; ☉: Fair; ●: Poor

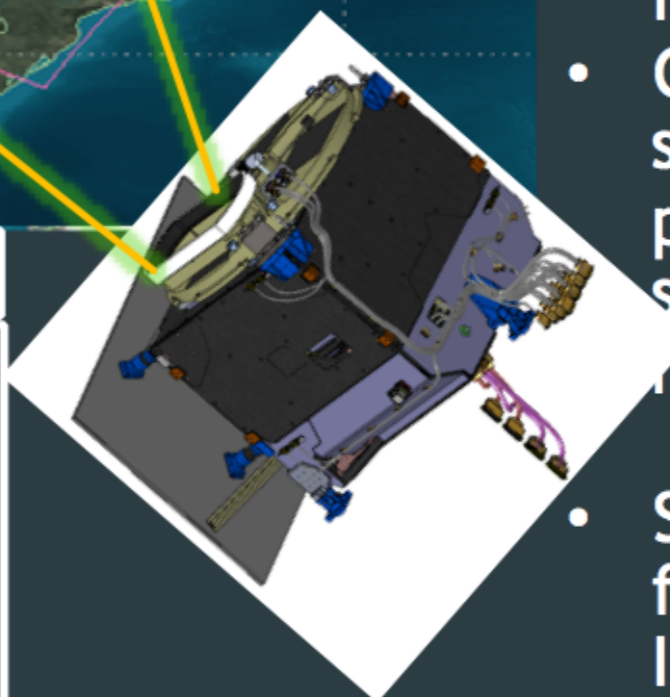
<sup>1</sup>Rascher [2007]; <sup>2</sup>Guanter et al. [2015]; <sup>3</sup>Joiner et al. [2013]; <sup>4</sup>Joiner, unpublished; <sup>5</sup>Frankenberg et al. [2014]; <sup>6</sup>Joiner et al. [2011, 2012]; <sup>7</sup>Miejer et al. [2014]; <sup>8</sup>Key et al. [2012]; <sup>9</sup>Buchwitz et al. [2013]

\*F: in formulation (not yet selected) | S: selected for launch | O: in orbit \*\* Global in extent, but sparse coverage (not complete) \*\*\* Far-red SIF only

# Geostationary: Stop-and-Stare Imaging Spectrometer



- Coverage - most of the highly productive regions of the Americas - including Amazon and agricultural areas.
- First dedicated, measurement of solar-induced fluorescence (SIF, both red and far-red) and the Photochemical Reflectance Index (PRI).
- Goals - direct measurement of seasonal and diurnal photosynthesis; detection of stress, and improvement of models.
- Synergistic parameters derived from GOES-R Advanced Baseline Imager
- More chances to see between the clouds.



## Performance Parameters

- 4.7 seconds per image and step
- 1521 images per 120 minute SE-NW scan  
Daily solar calibration.
- Can focus in on selected areas for intensive study.
- 490-780 nm spectral range 0.3 nm FWHH  
signal to noise >1000 (in stop-n-stare mode)
- Instrument optimized for solar-induced fluorescence measurements.



**Greg Asner**



**Scott Denning**



**Jim Randerson**



**Ruth de Freis**



**Chris Still**



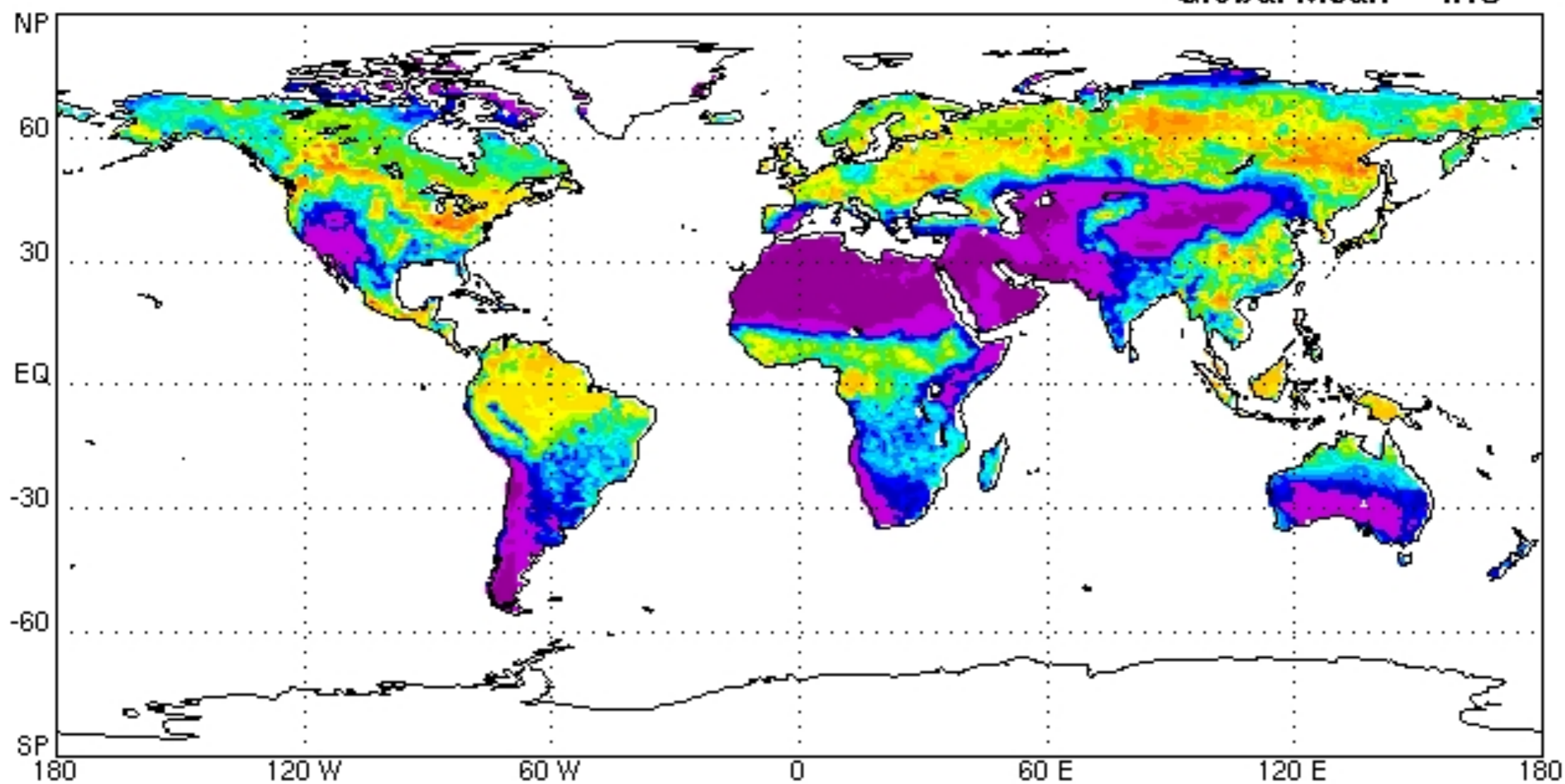
**Ian Baker**

Nov 12, 2007

## July 2000 CANOPY NET PHOTOSYNTHESIS

$\mu\text{moles/m}^2/\text{s}$

Global Mean = 4.13



0.05 0.80 1.55 2.30 3.05 3.80 4.55 5.30 6.05 6.80 7.55 8.30 9.05 9.80 10.55 11.30 12.05 12.80

To summarize:

Breakthroughs aren't obvious until they happen.

The pieces that ultimately fit together are the key to breakthroughs; we need to be looking for them, and we need cultivate them.

In light of the Decadal Survey, I'd like to make the point that the NASA centers are a tremendous resource for this “scientific potential energy”

flipping the switches of the universe





# That's how we got here. Now, where are we going?

## Several speakers have already mentioned fluorescence

### Key discoveries:

Plascyk, J. A. (1975). The MK II Fraunhofer Line Discriminator (FLD-II) for Airborne and Orbital Remote Sensing of Solar-Stimulated Luminescence. *Optical Engineering*, 14(4), 339–0. <http://doi.org/10.1117/12.7971842>

Perkin-Elmer &  
Wollops

Guanter, L., Alonso, L., Gómez-Chova, L., Amorós-López, J., Vila, J., & Moreno, J. (2007). Estimation of solar-induced vegetation fluorescence from space measurements. *Geophysical Research Letters*, 34(8), L08401. <http://doi.org/10.1029/2007GL029289>

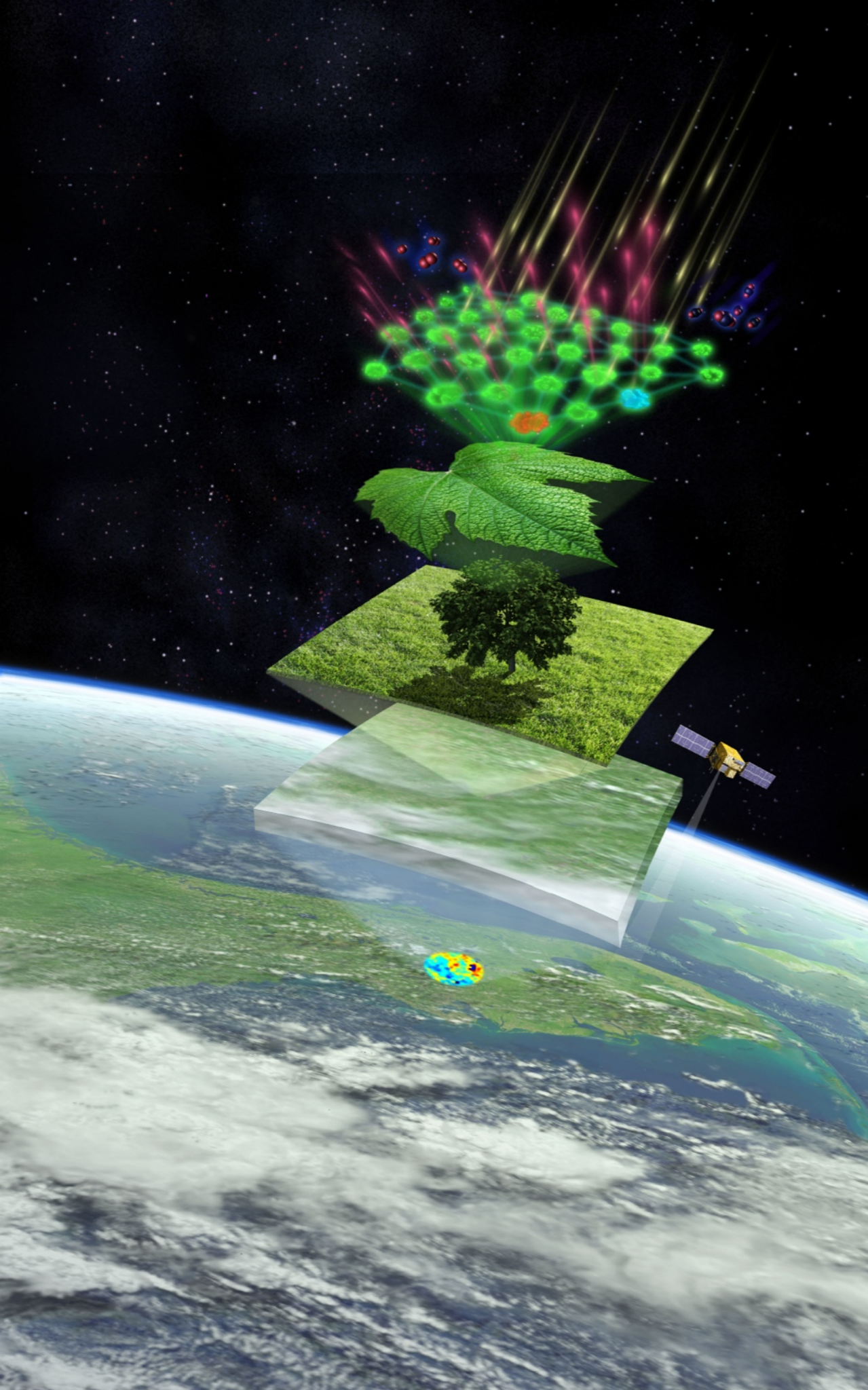
ESA

Frankenberg, C., Butz, A., & Toon, G. C. (2011). Disentangling chlorophyll fluorescence from atmospheric scattering effects in O 2A-band spectra of reflected sun-light. *GEOPHYSICAL RESEARCH LETTERS*, 38(3), L03801. doi: 10.1029/2010GL045896

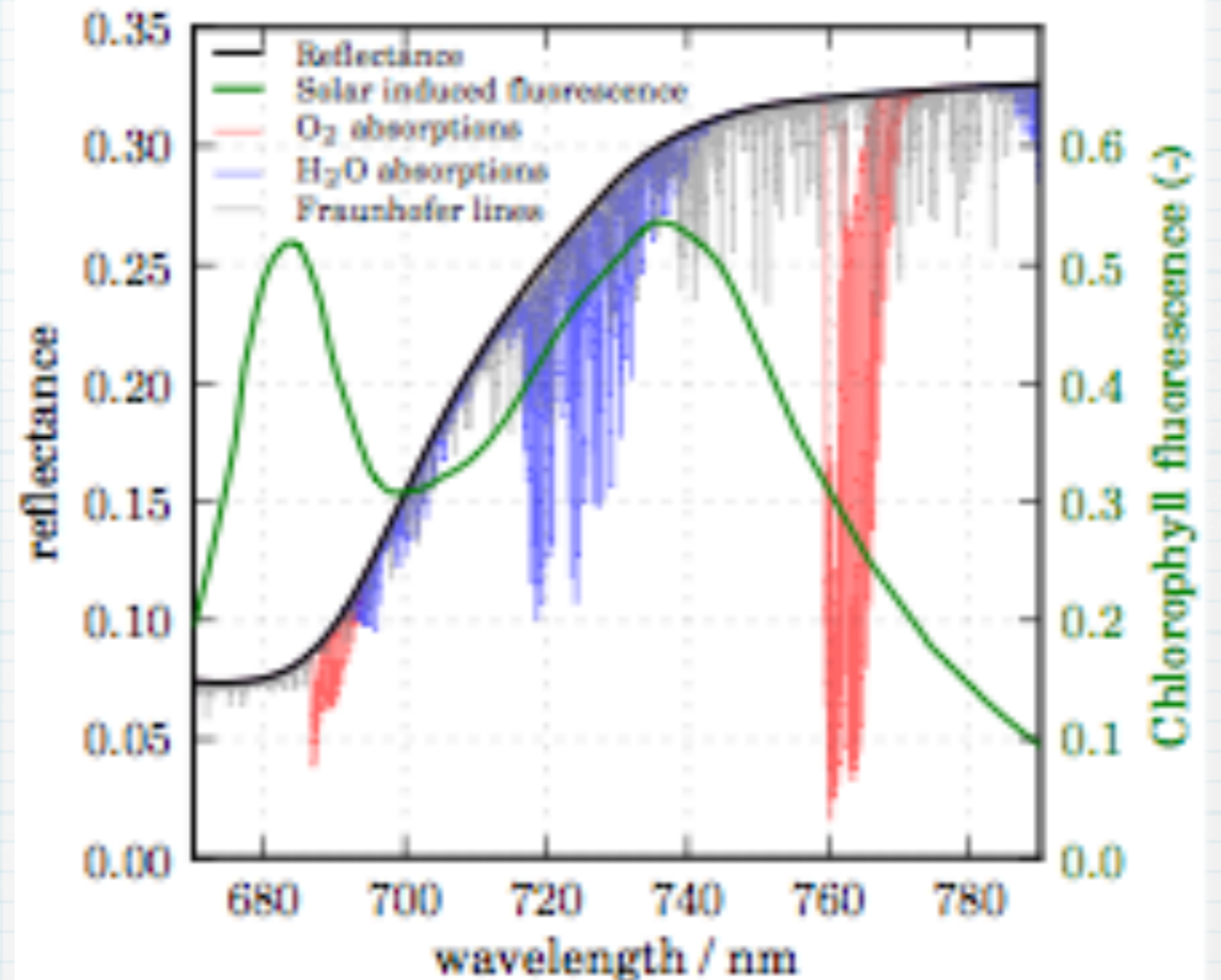
JPL

Joiner, J., Yoshida, Y., Vasilkov, A. P., Yoshida, Y., Corp, L. A., & Middleton, E. M. (2011). First observations of global and seasonal terrestrial chlorophyll fluorescence from space. *Biogeosciences*, 8(3), 637–651. doi:10.5194/bg-8-637-2011

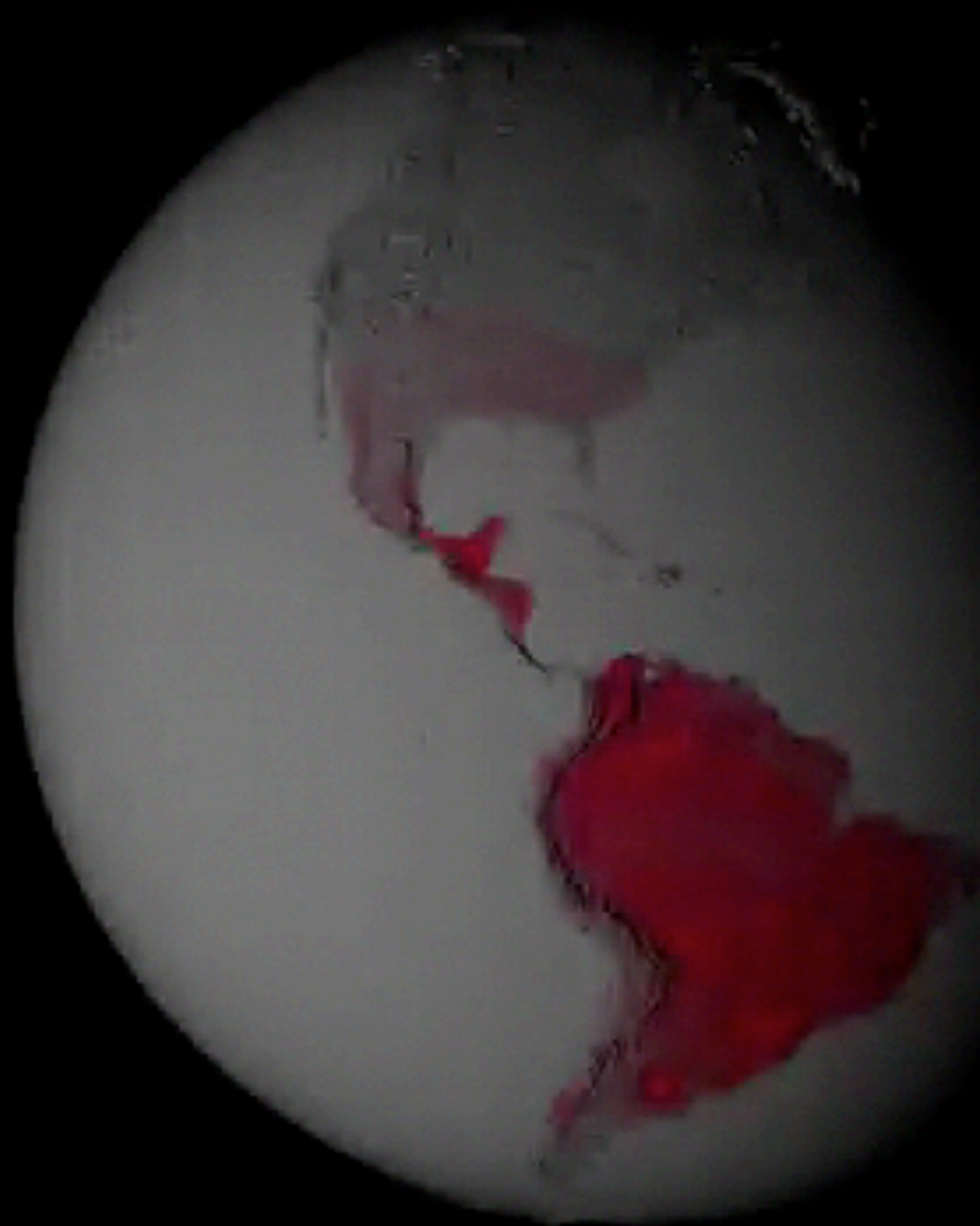
GSFC



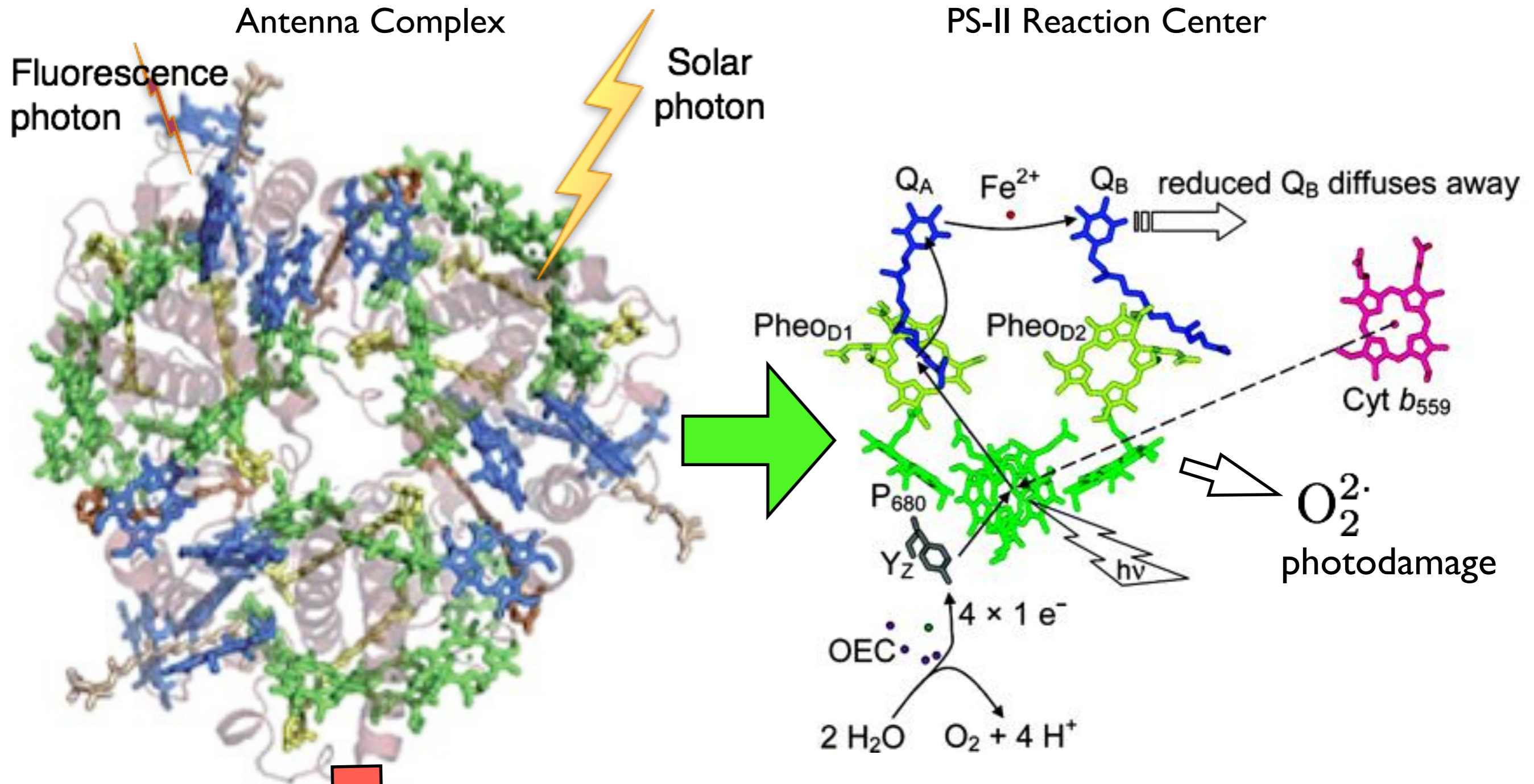
- Fluorescence from terrestrial plants is difficult to measure



- In-filling of lines in the solar spectrum can be used to distinguish fluorescence from reflected light.
- It takes a special high resolution spectrometer. So far it has been accomplished with 5 satellites



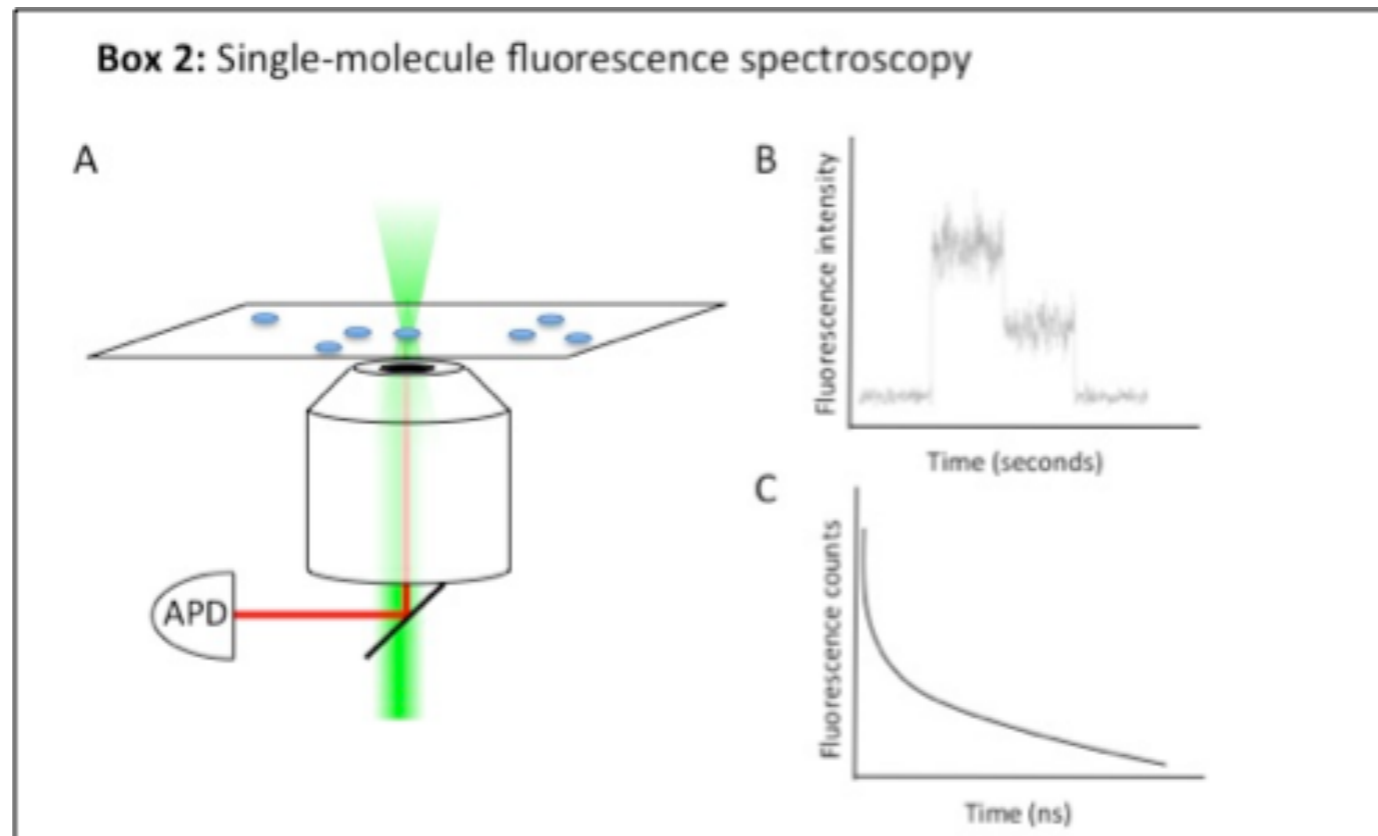
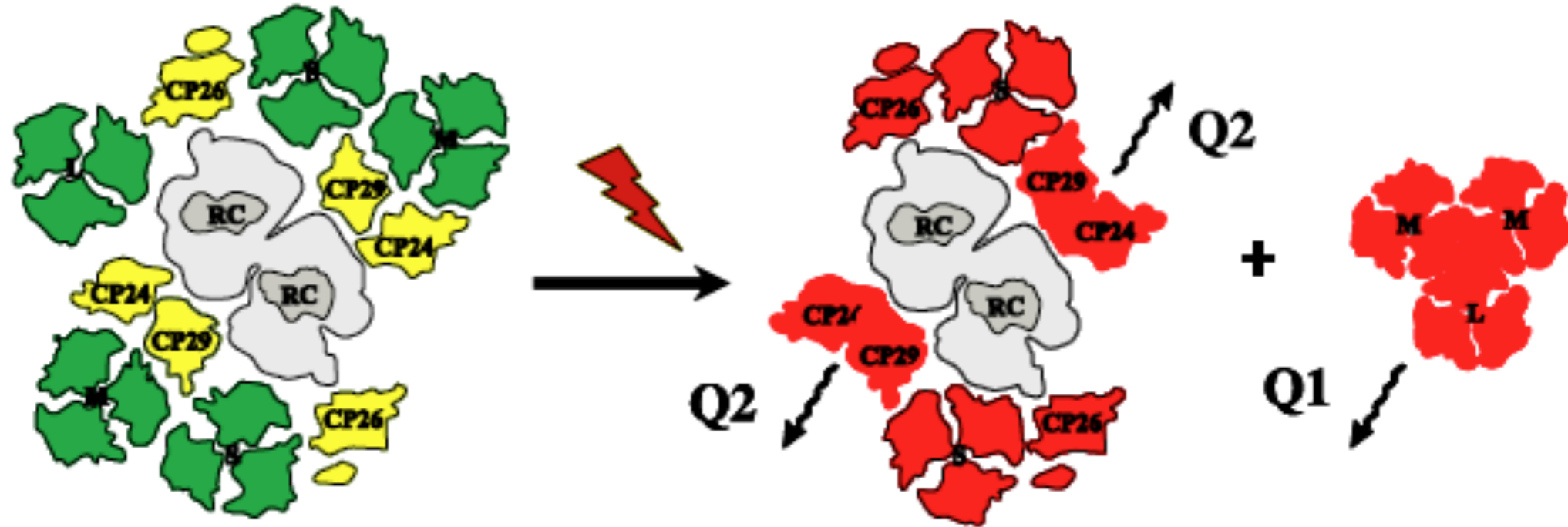
# Fluorescence in Photosynthesis Research



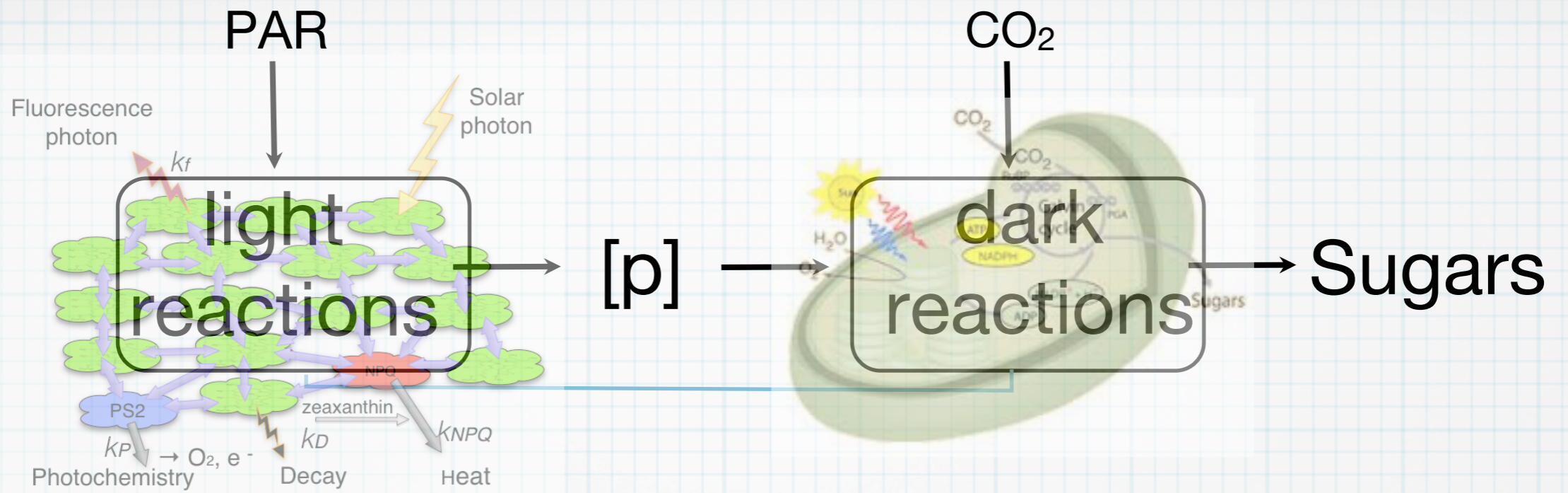
Non-photochemical  
Quenching

Fleming, G. R., Schlau-Cohen, G. S., Amarnath, K., & Zaks, J. (2012). Design principles of photosynthetic light-harvesting. *Faraday Discussions*, 155, 27. <http://doi.org/10.1039/c1fd00078k>

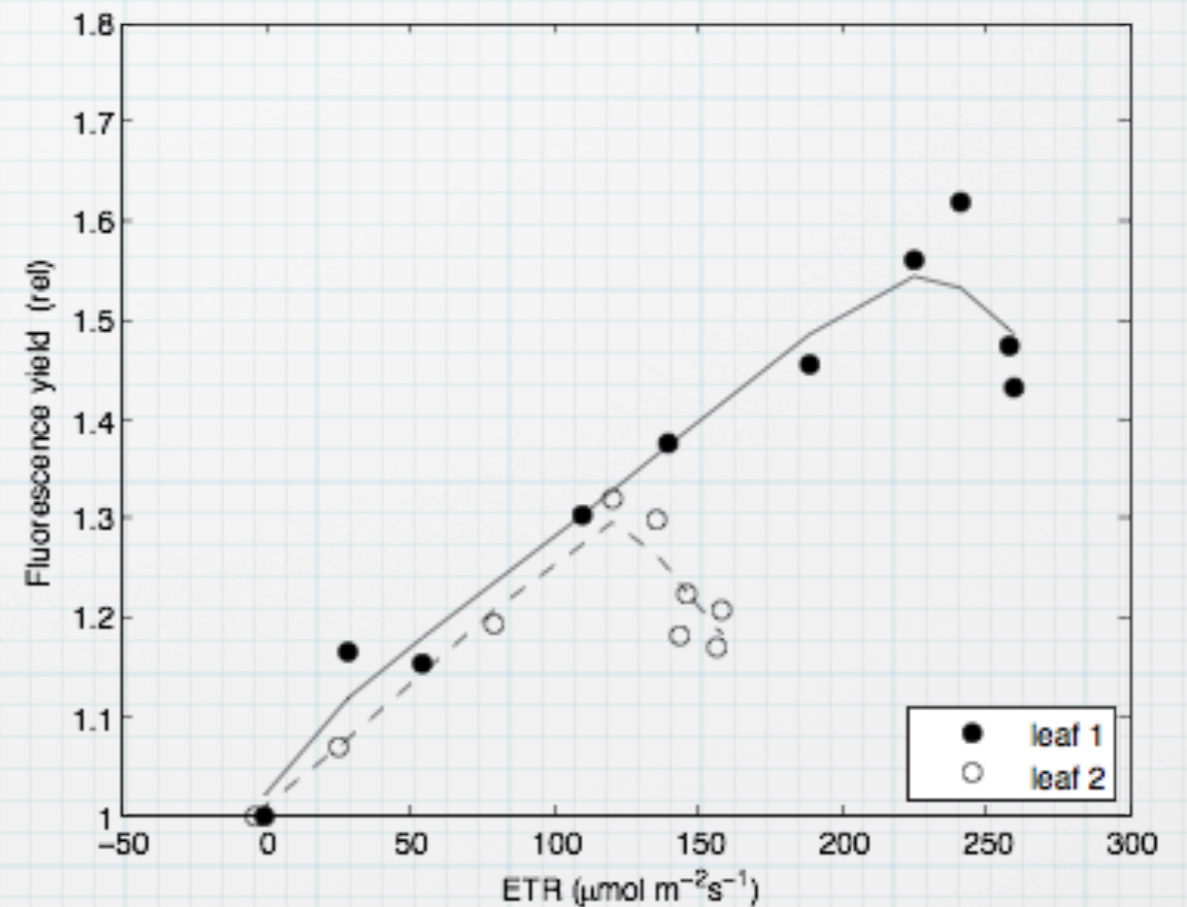
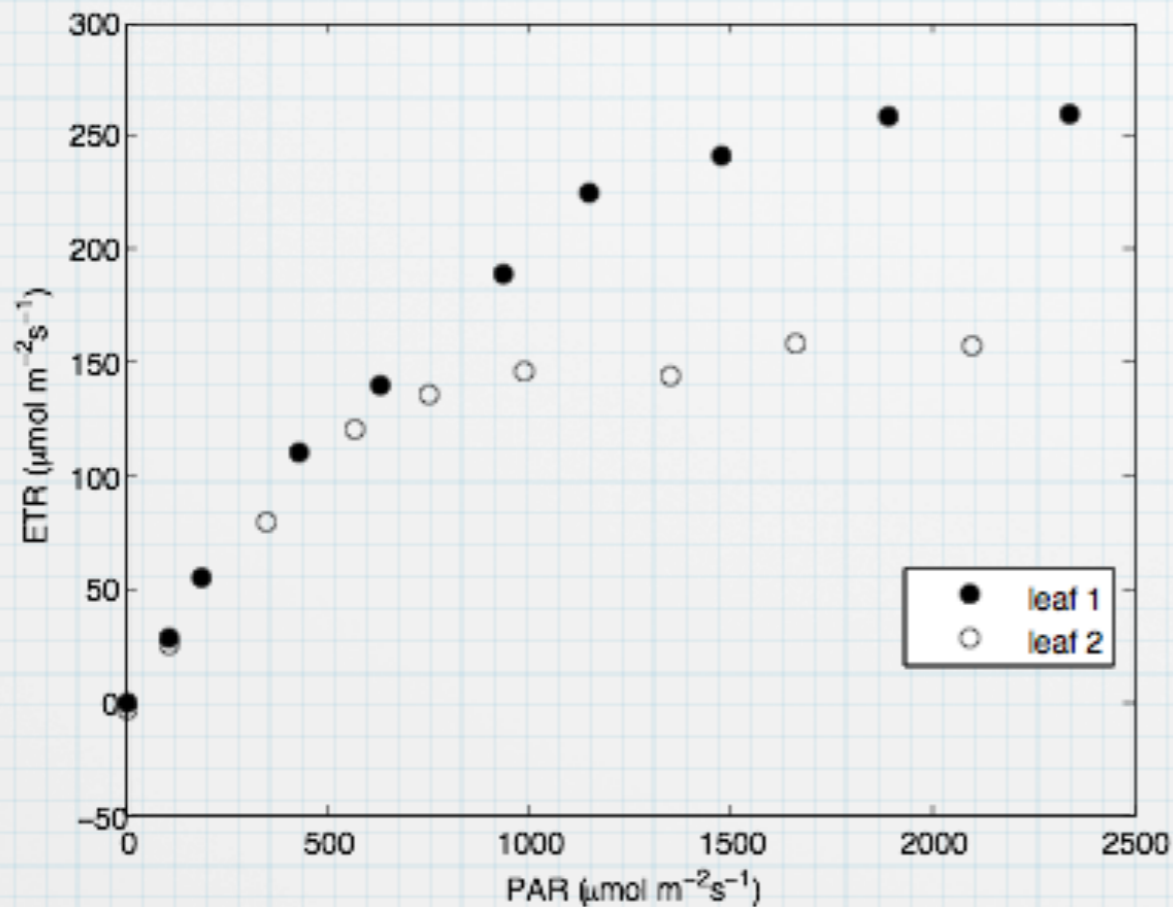
# Non Photochemical Quenching



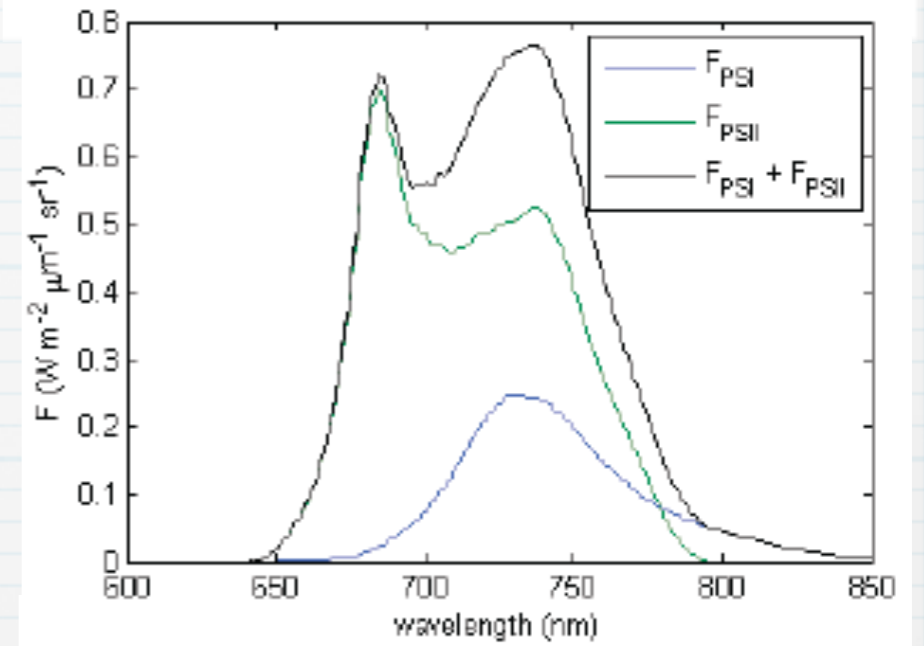
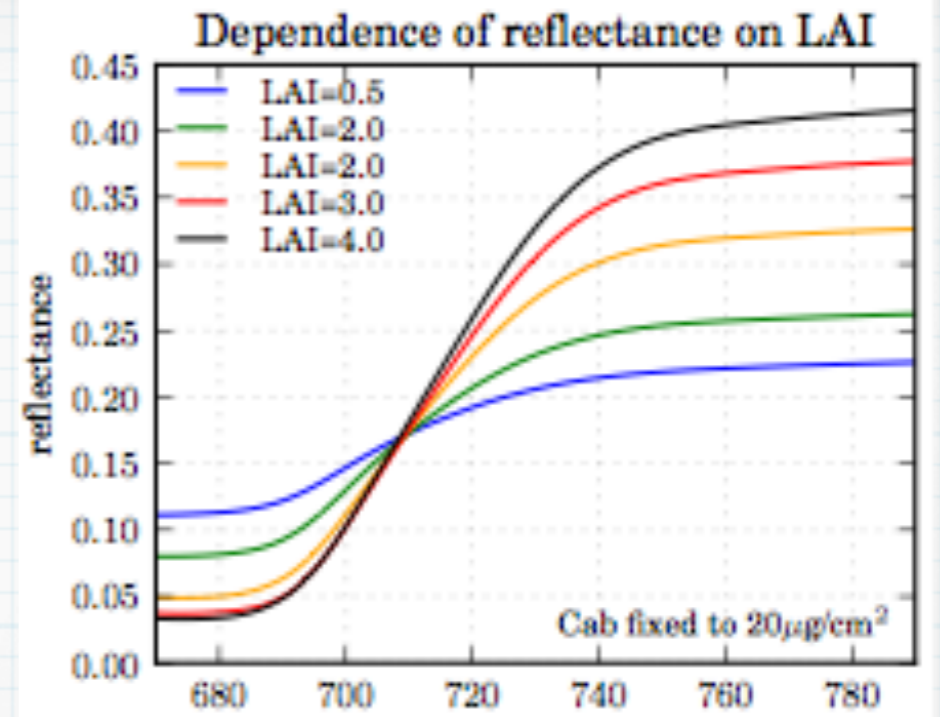
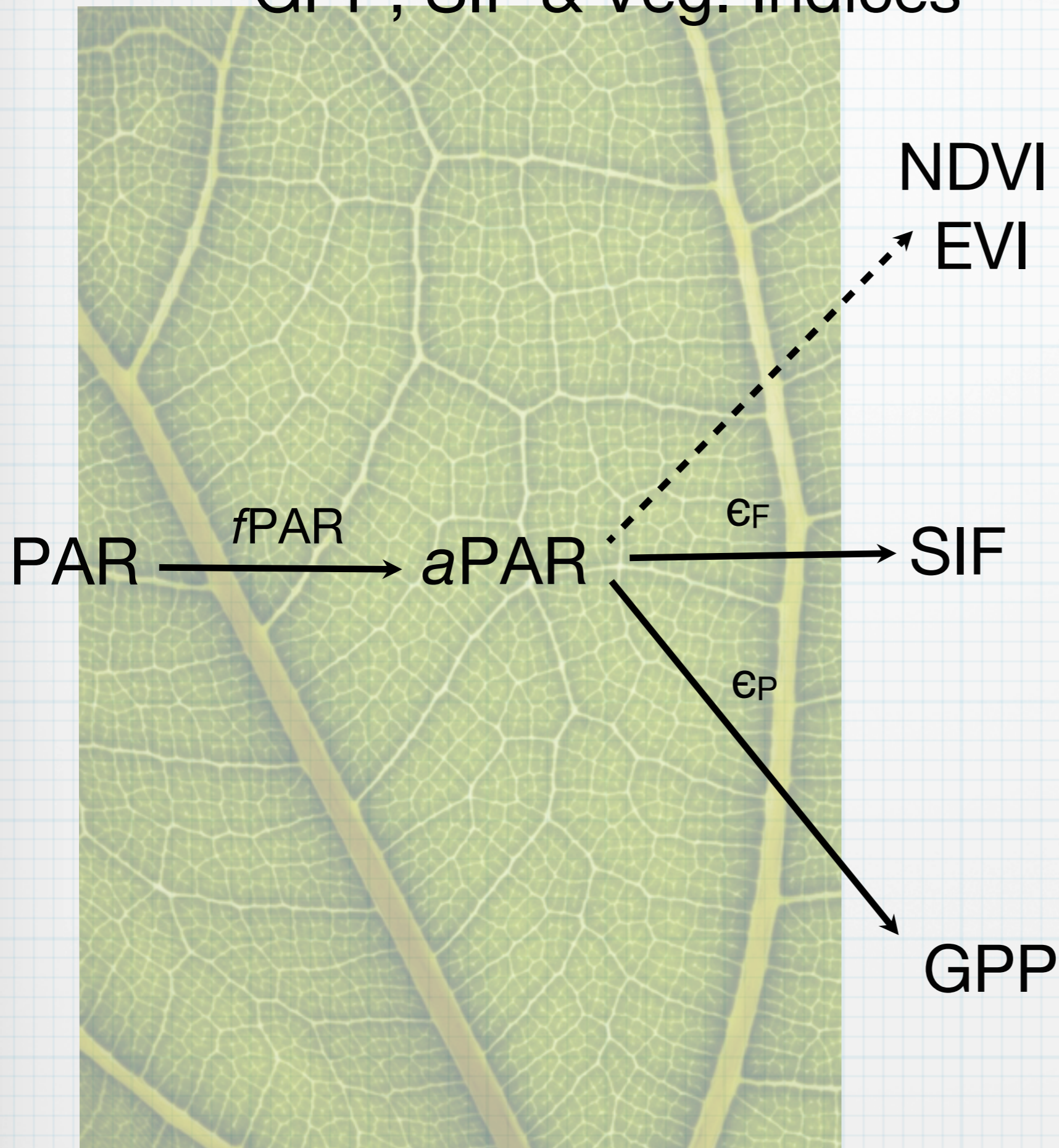
Schlau-Cohen, G. S., Bockenhauer, S., Wang, Q., & Moerner, W. E. (2014). Single-molecule spectroscopy of photosynthetic proteins in solution: exploration of structure–function relationships. *Chemical Science*, 5(8), 2933. <http://doi.org/10.1039/c4sc00582a>



## Fluorescence vs. the electron transport rate (ETR)



# GPP, SIF & Veg. Indices



# GPP and Solar Induced Fluorescence (SIF)

$$aPAR_R = f(\text{NDVI}) \times \text{PAR}$$

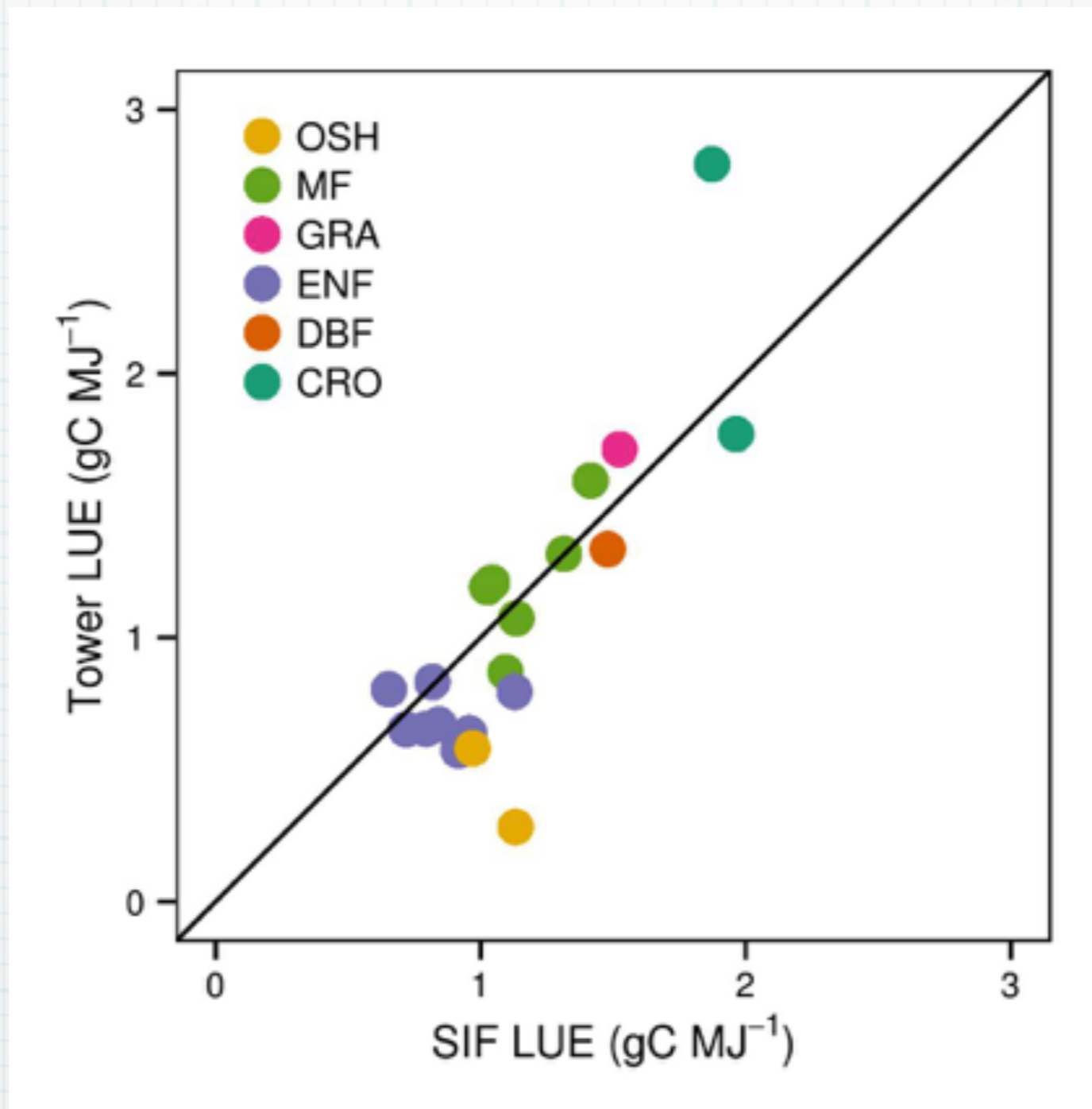
$$\text{GPP} = aPAR \times \epsilon_P$$

$$\text{SIF} = aPAR \times \epsilon_F$$

$$\text{GPP} = \text{SIF} \times \frac{\epsilon_P}{\epsilon_F}$$



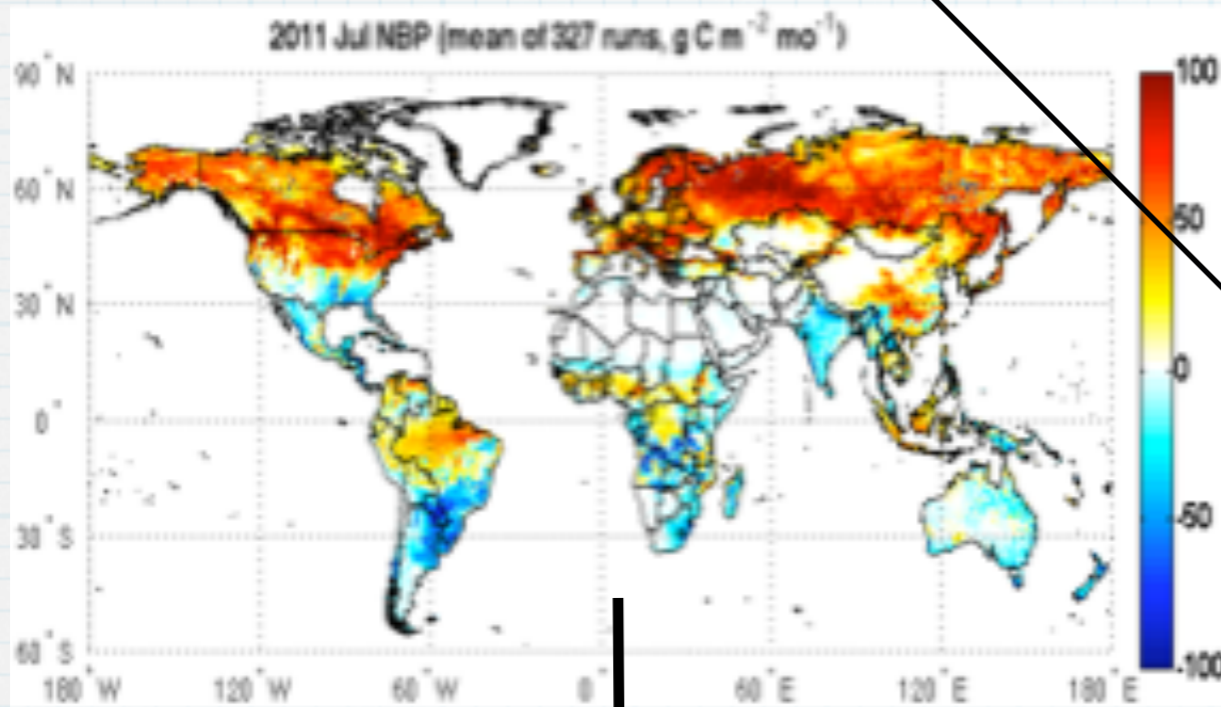
# $\epsilon_P$ vs. $\epsilon_F$ at Flux Towers



Grayson Badgley, John Kimball et al.

# CASA, carbon cycle model

G. James Collatz, Joanna Joiner, Stephan R Kawa et al.

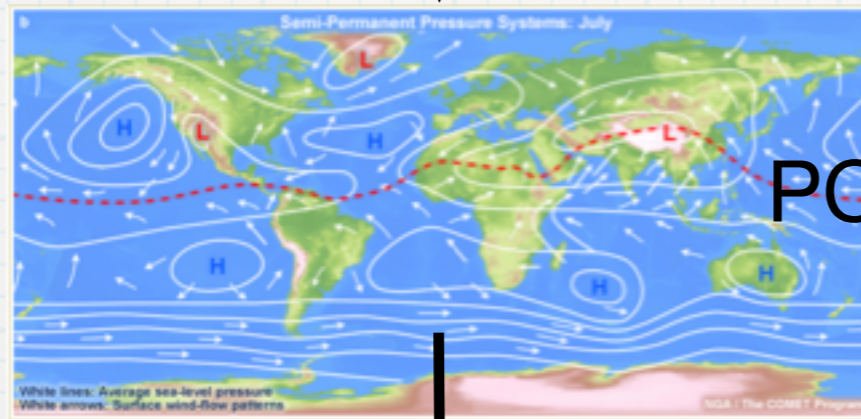


$$NPP = PAR \cdot FPAR \cdot LUE$$

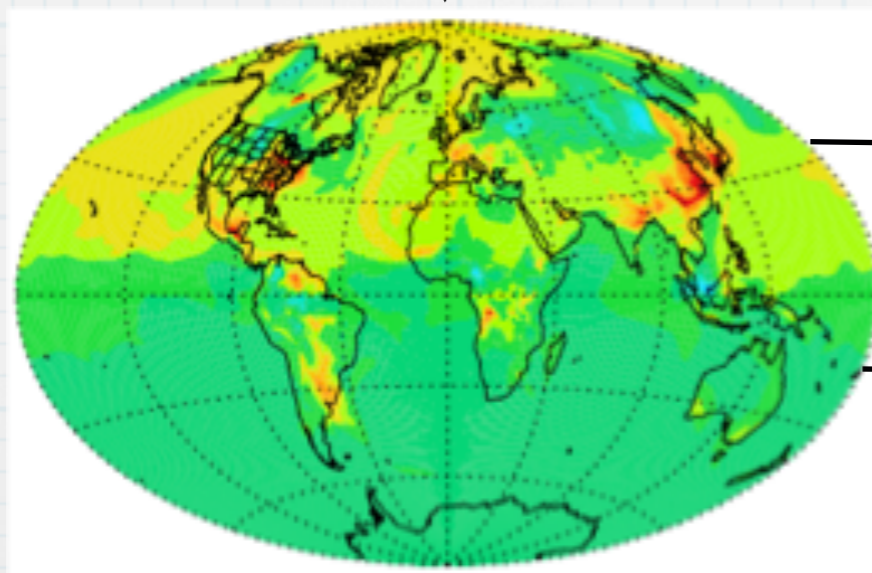
$$\text{where } LUE = f(T, P/PET)$$

$$NPP = k \cdot SIF$$

$$\text{where : } k = \frac{\overline{NPP}_{max}}{\overline{SIF}_{max}}$$



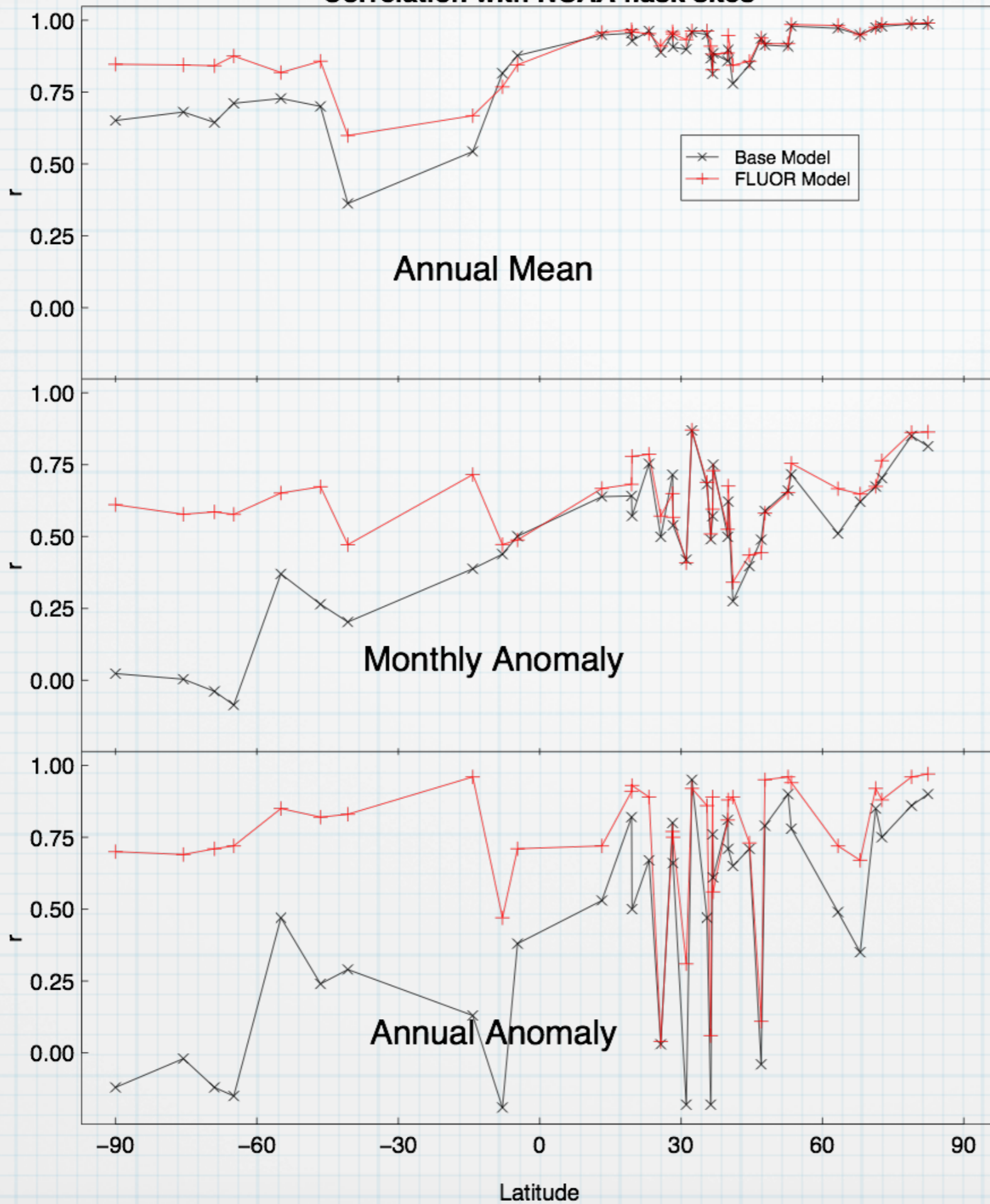
PCTM, atmos. transport model



Baseline model  $[CO_2]_{i,j}$

Flour. model  $[CO_2]_{i,j}$

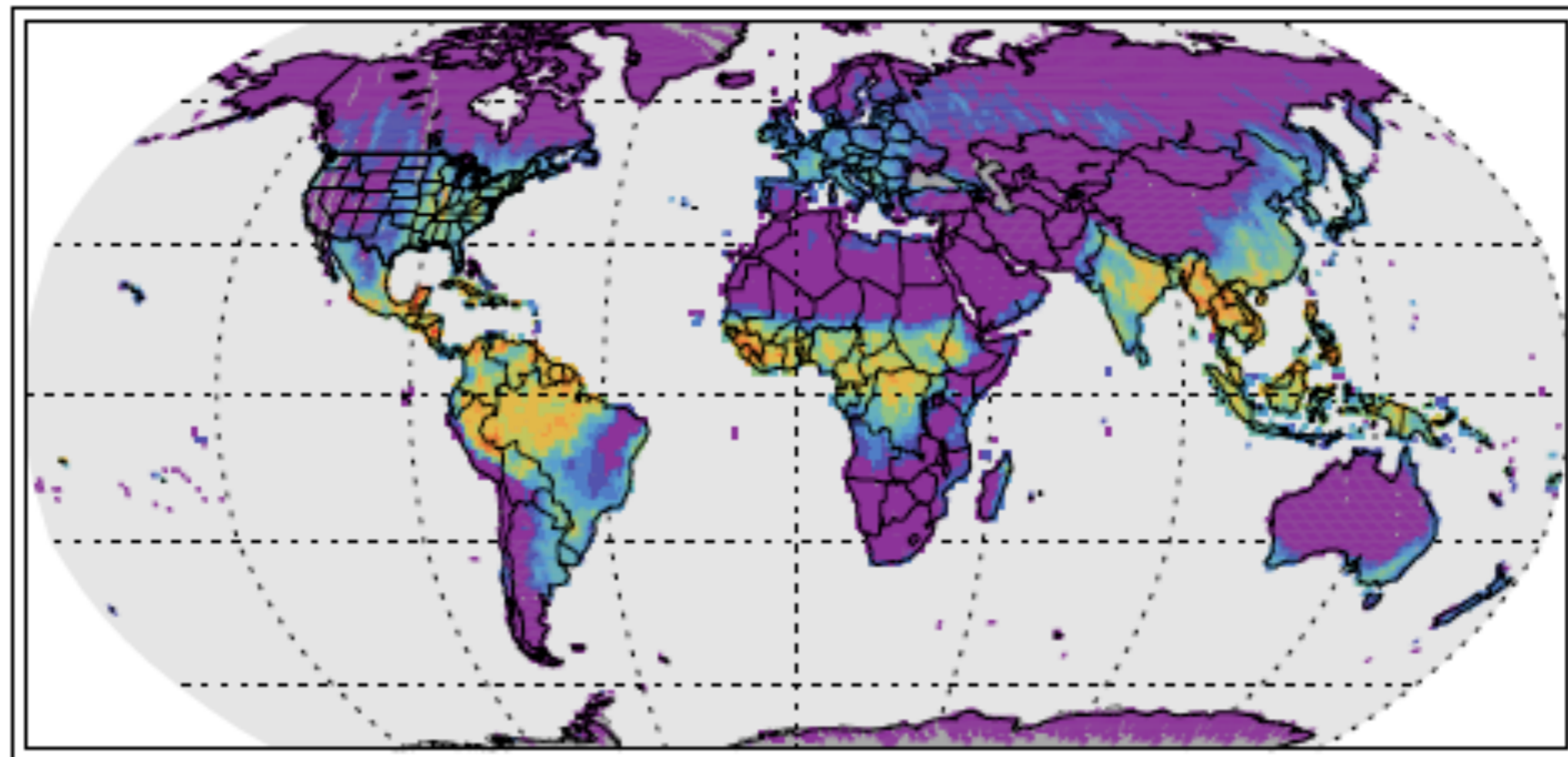
### Correlation with NOAA flask sites



Jim Collatz

# 3-MONTH OCO-2 AVERAGE (ALL MODES)

OCO-2 Solar-Induced Fluorescence Aug-Oct 2014



SIF / ( $\text{W m}^{-2} \text{ micron}^{-1} \text{ sr}^{-1}$ )

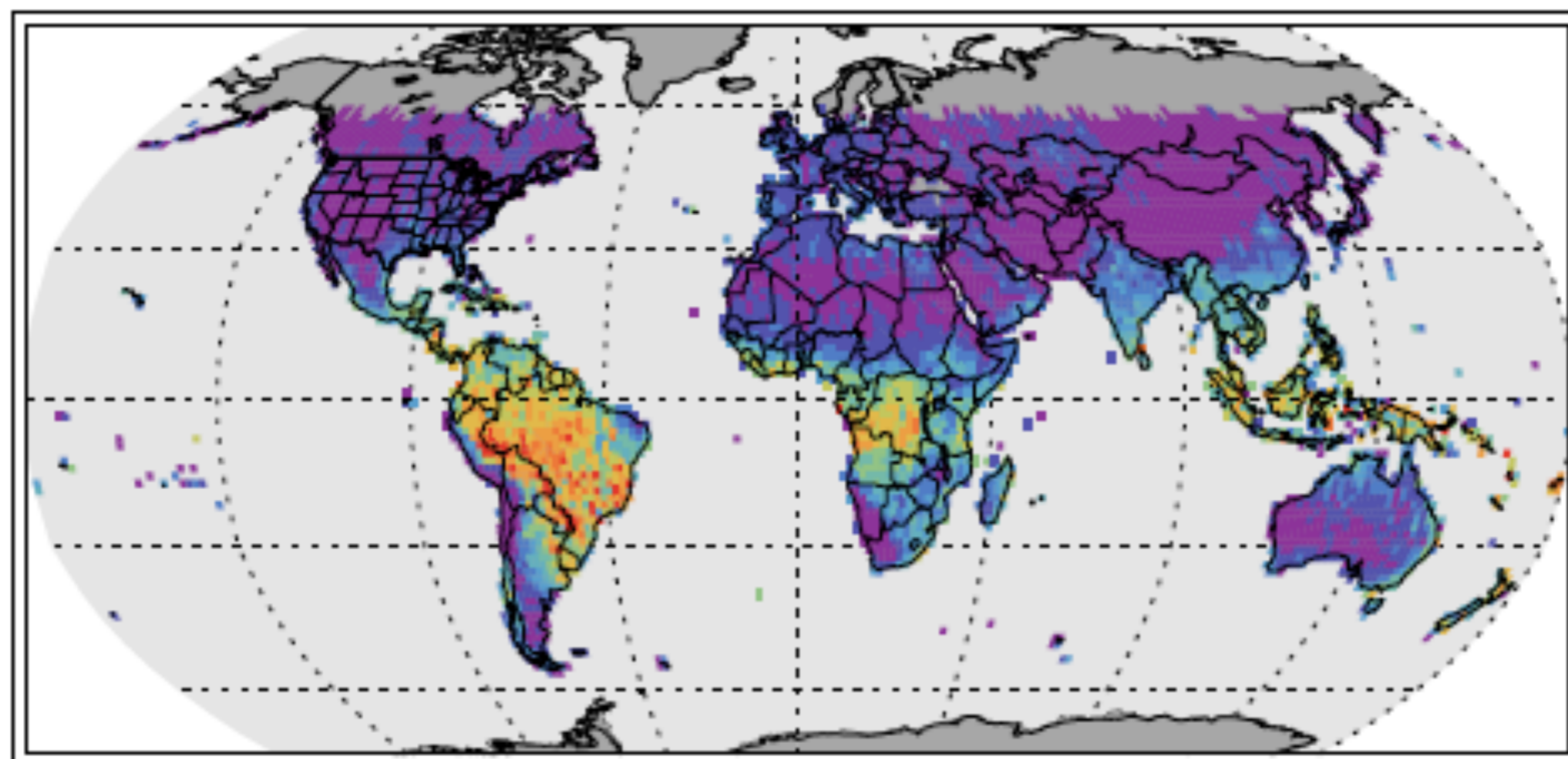
0.00 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1.00 1.10 1.20

still tentative...

Christian Frankenberg

# LATEST B5000 DATASET (NOV/DEC. 2014)

OCO2 B5000x4

 $SIF / (W m^{-2} micron^{-1} sr^{-1})$ 

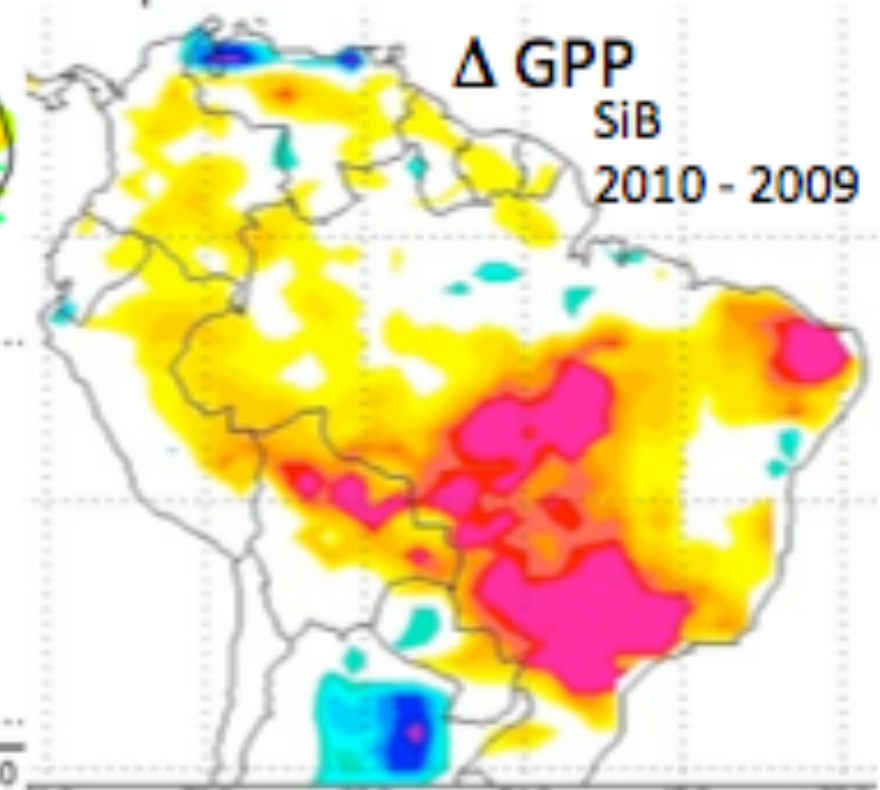
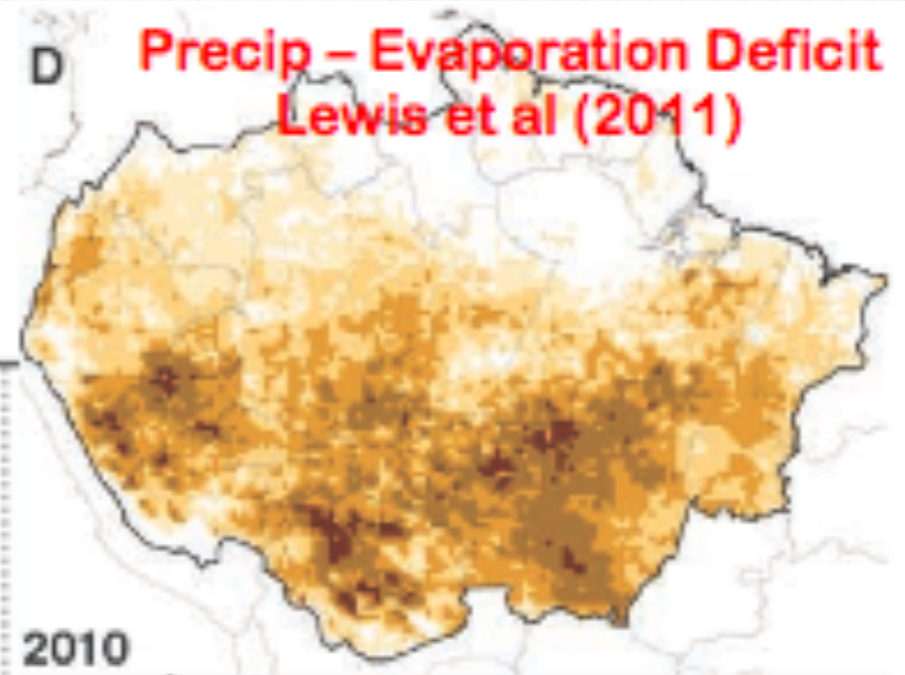
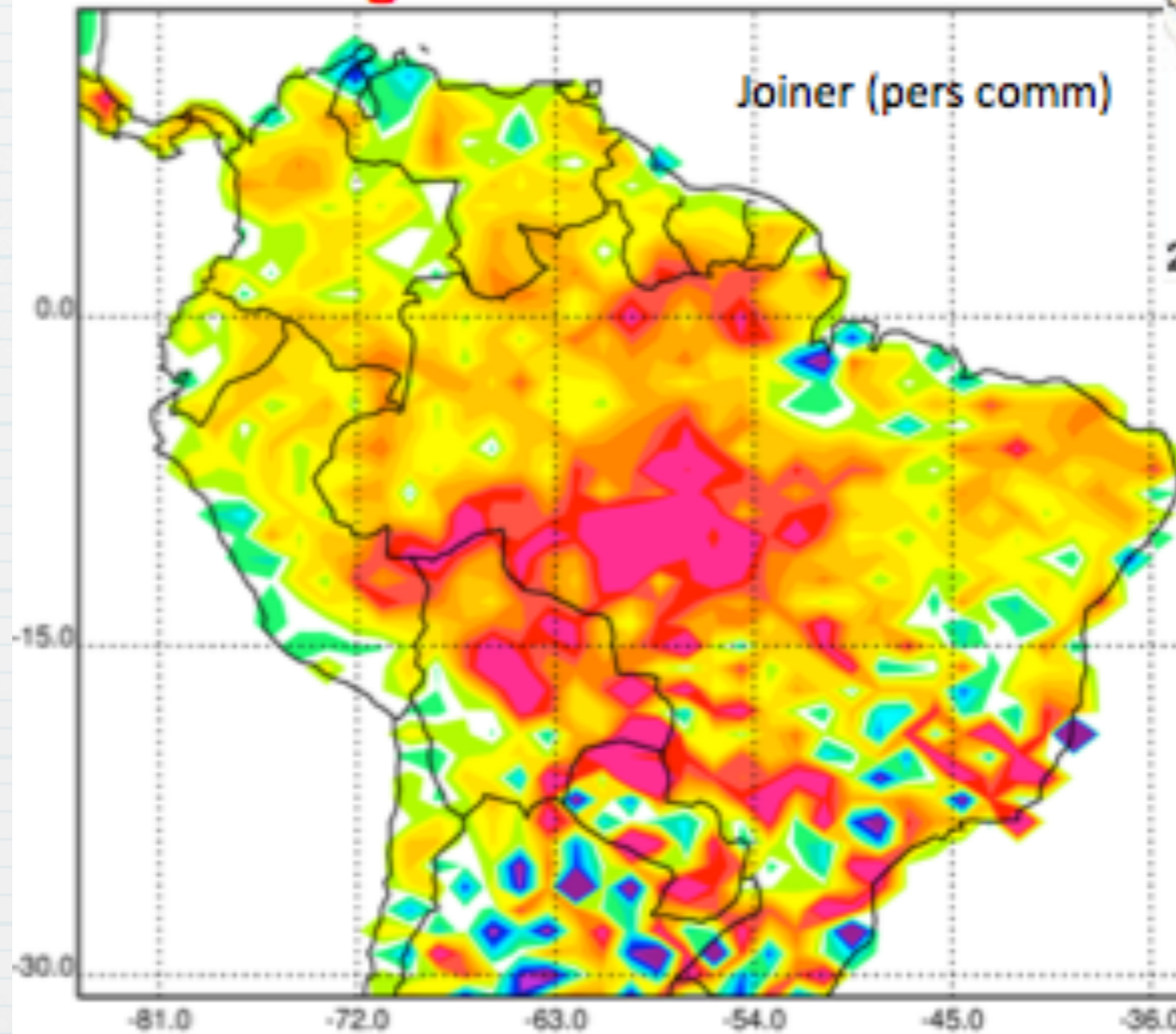
0.00 0.12 0.25 0.38 0.50 0.62 0.75 0.88 1.00 1.12 1.25 1.38 1.50

still tentative...

# Fluorescence Responds to Drought

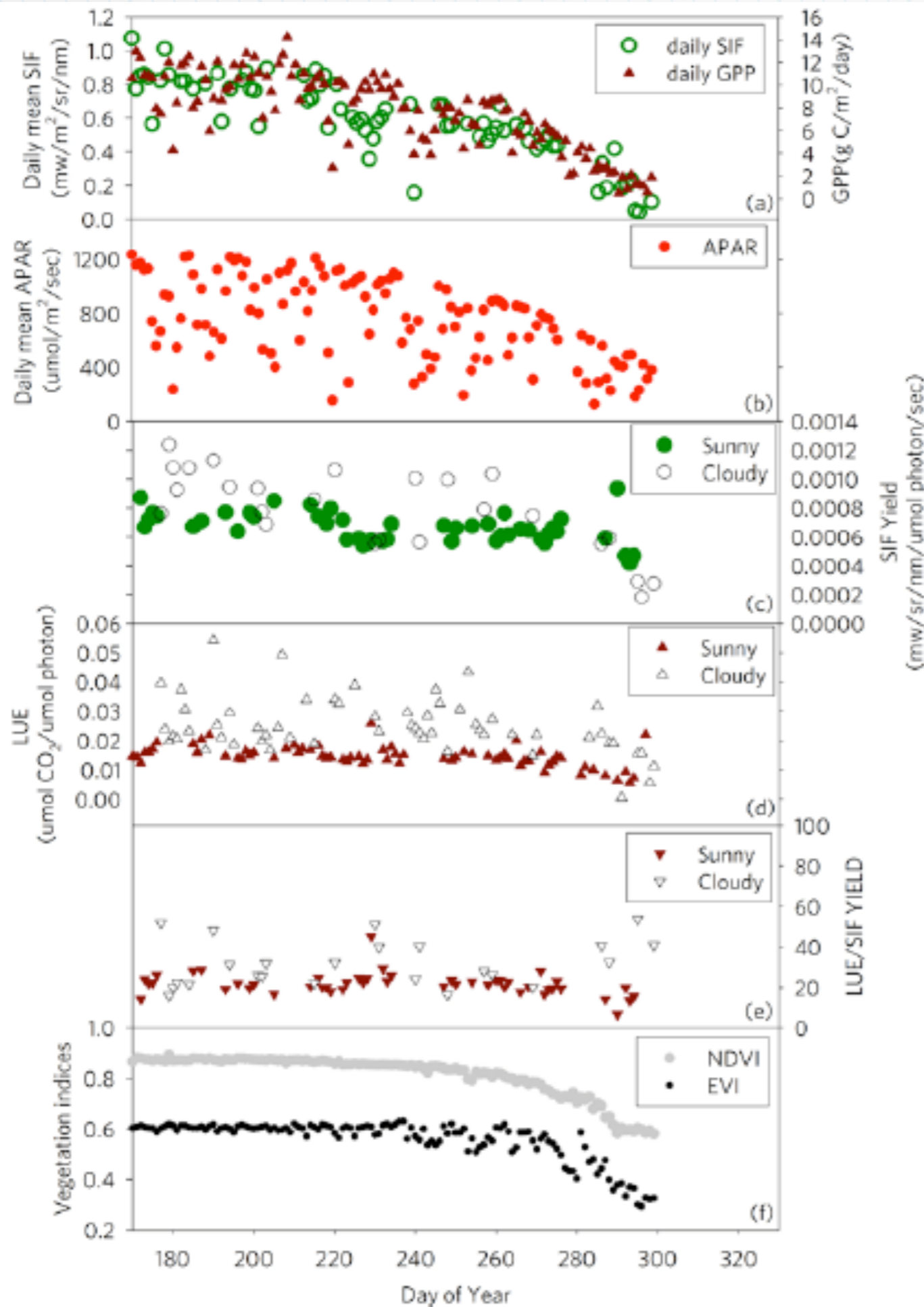
## 2010 Amazon Drought

Change in SIF 2010 - 2009



Ian Baker, Joanna Joiner & Scott Denning

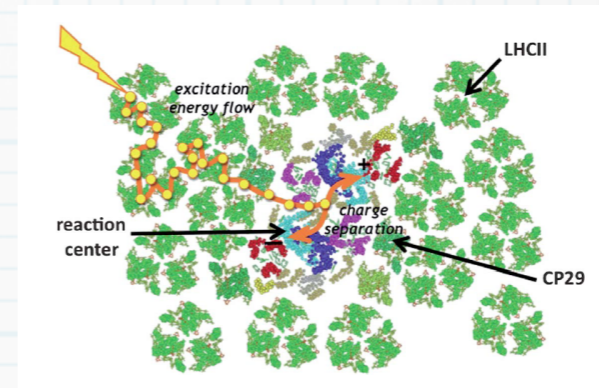
# Studies of fluorescence (SIF) at HF flux tower.



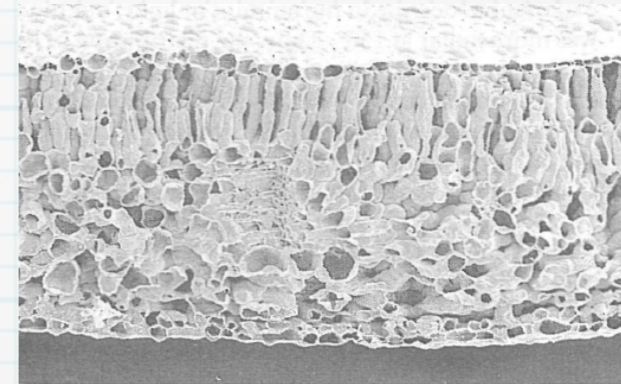
Yang, X., Tang, J., Mustard, J. F., Lee, J.-E., Rossini, M., Joiner, J., et al. (2015). Solar-induced chlorophyll fluorescence correlates with canopy photosynthesis on diurnal and seasonal scales in a temperate deciduous forest. *Geophysical Research Letters*, n/a–n/a. <http://doi.org/10.1002/2015GL063201>

# SIF

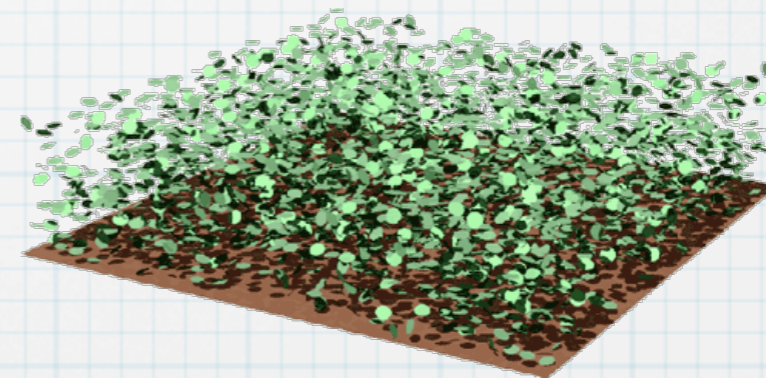
We need to consider hierarchy of scales



chloroplast  
membrane

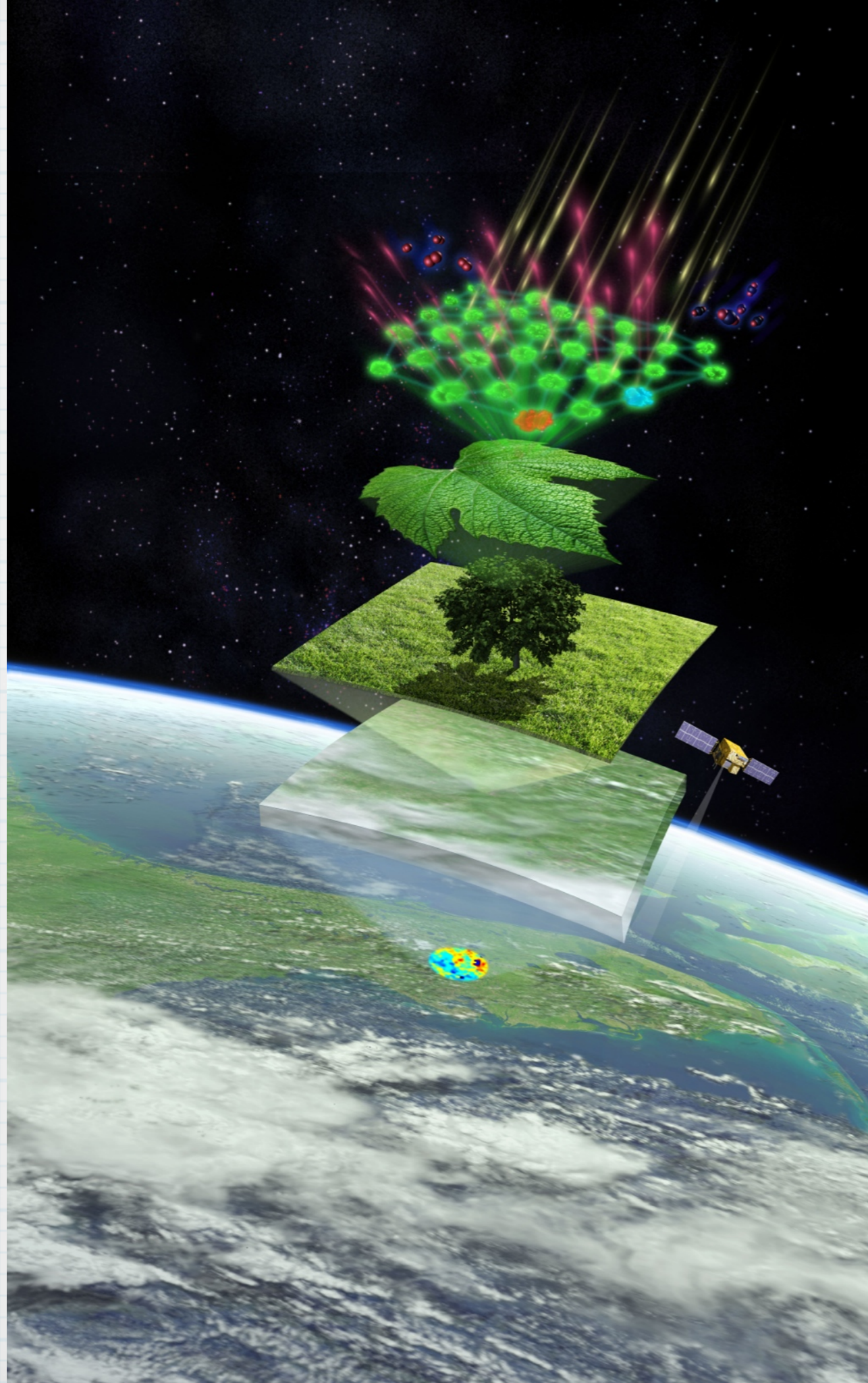


leaf



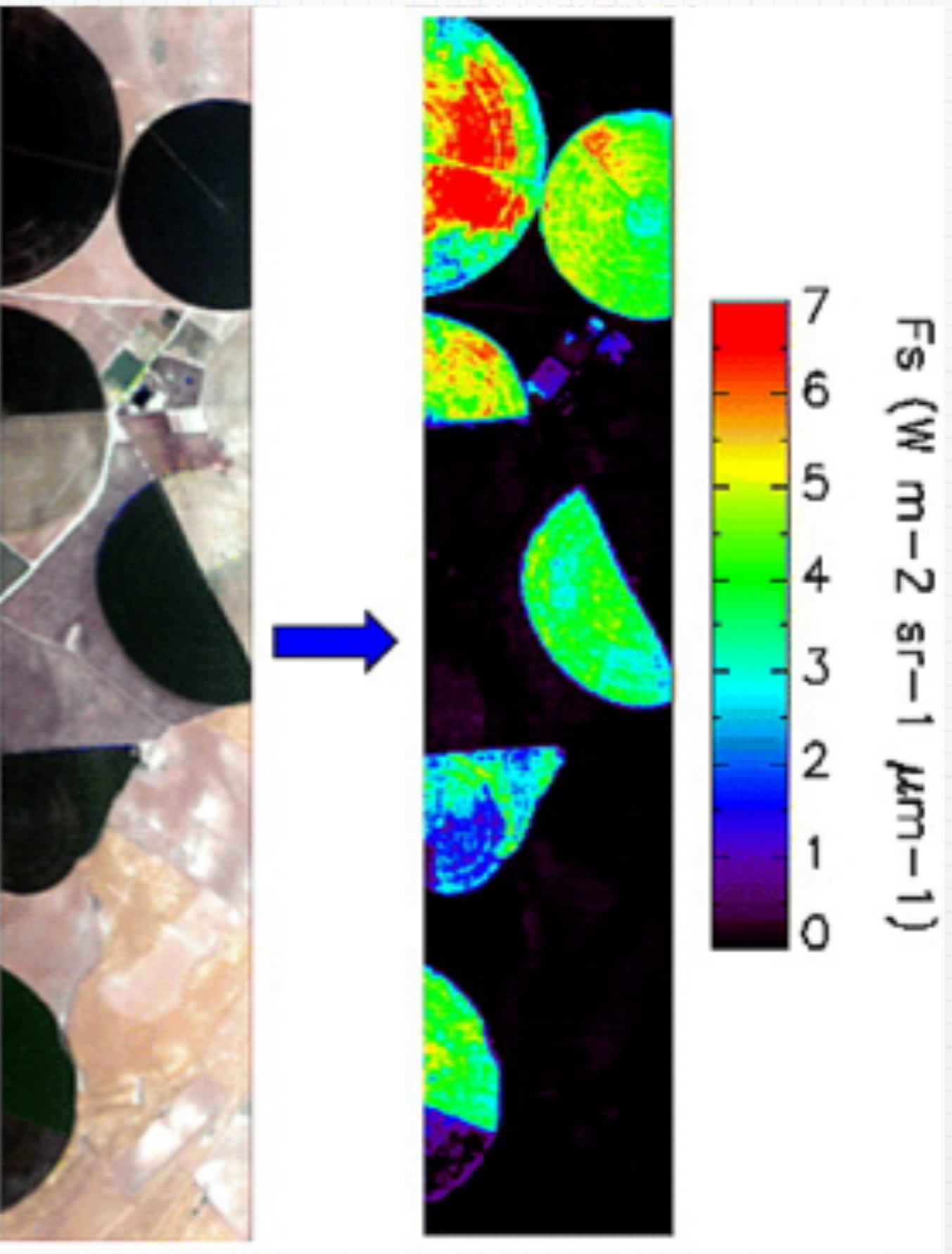
canopy

but understanding the mechanism is  
a critical first step.

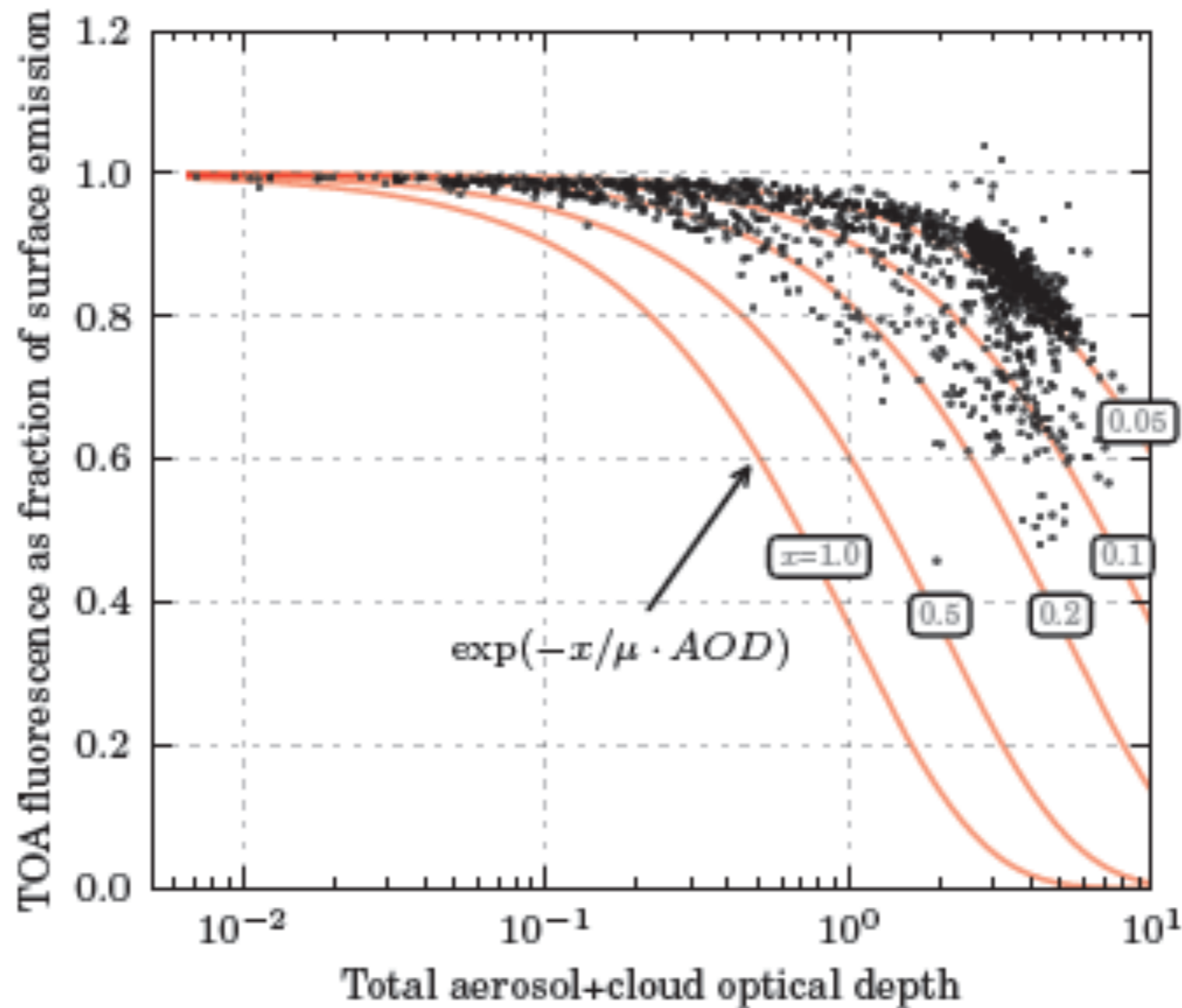




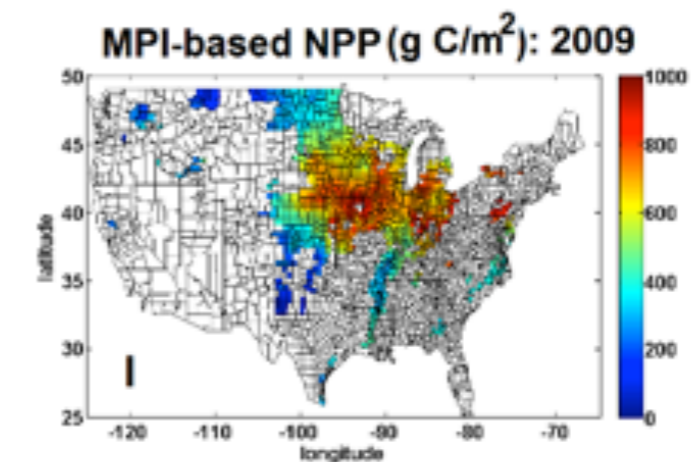
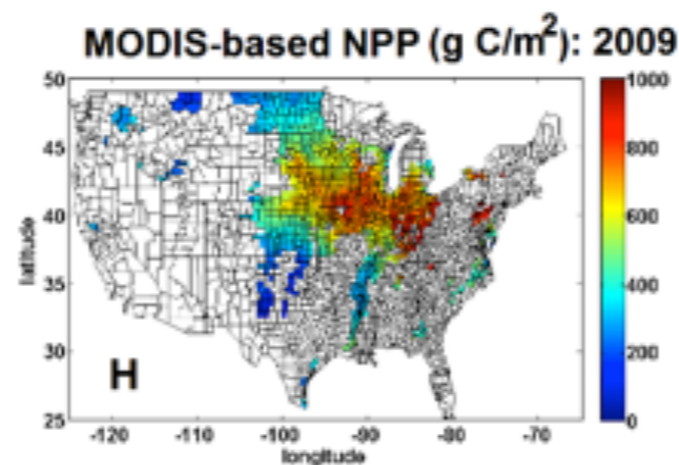
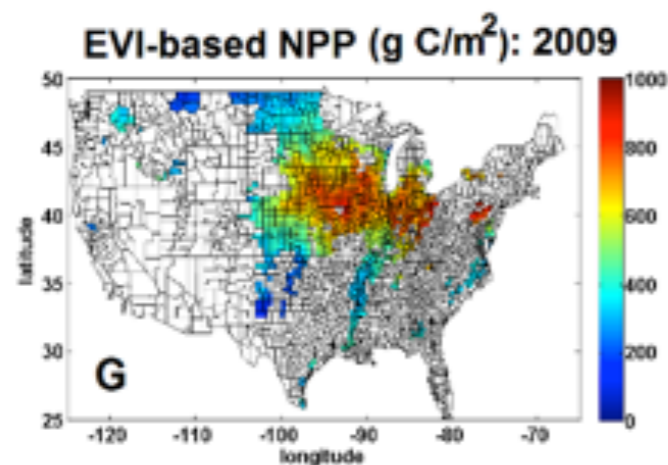
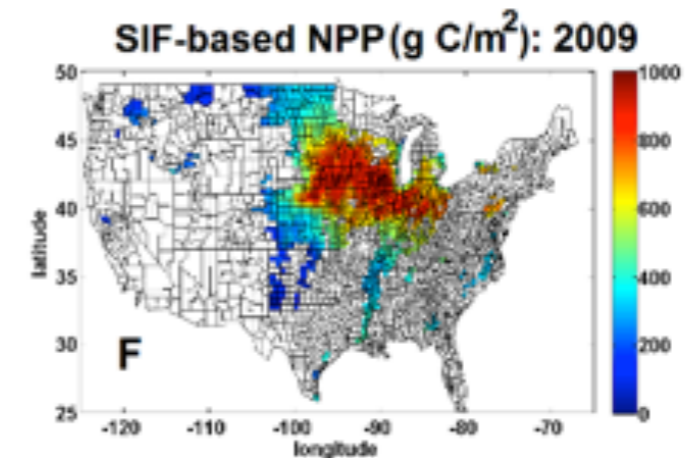
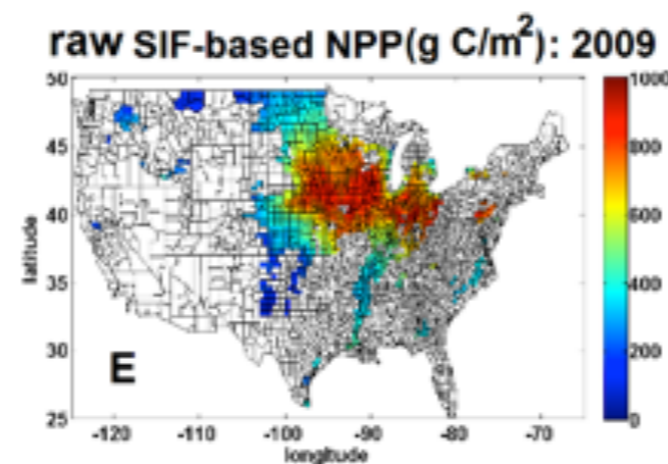
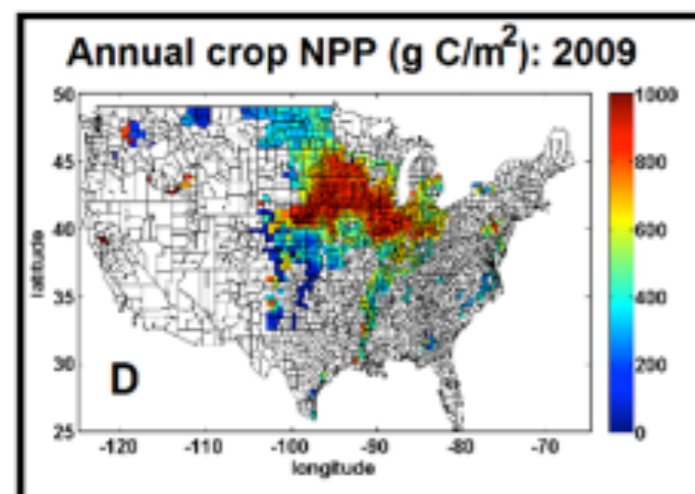
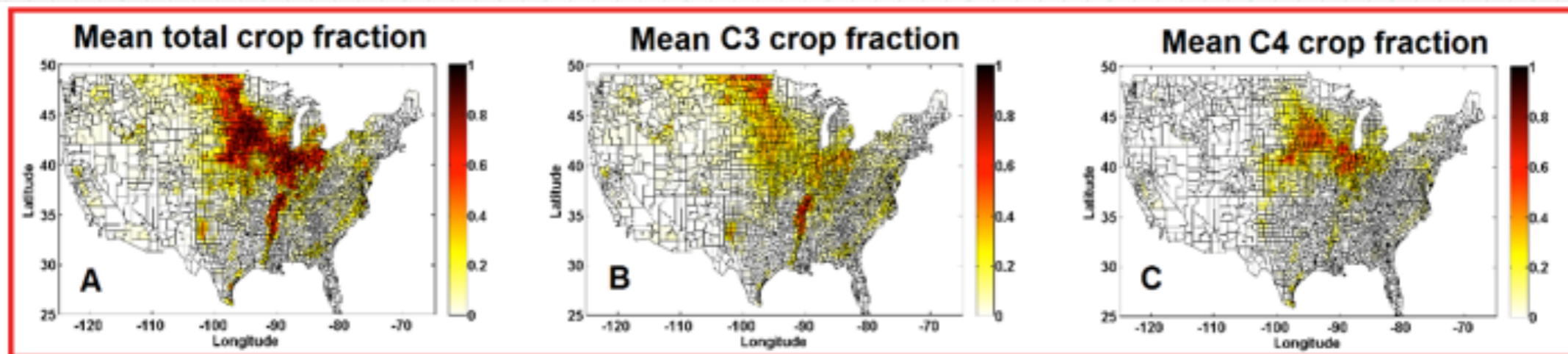
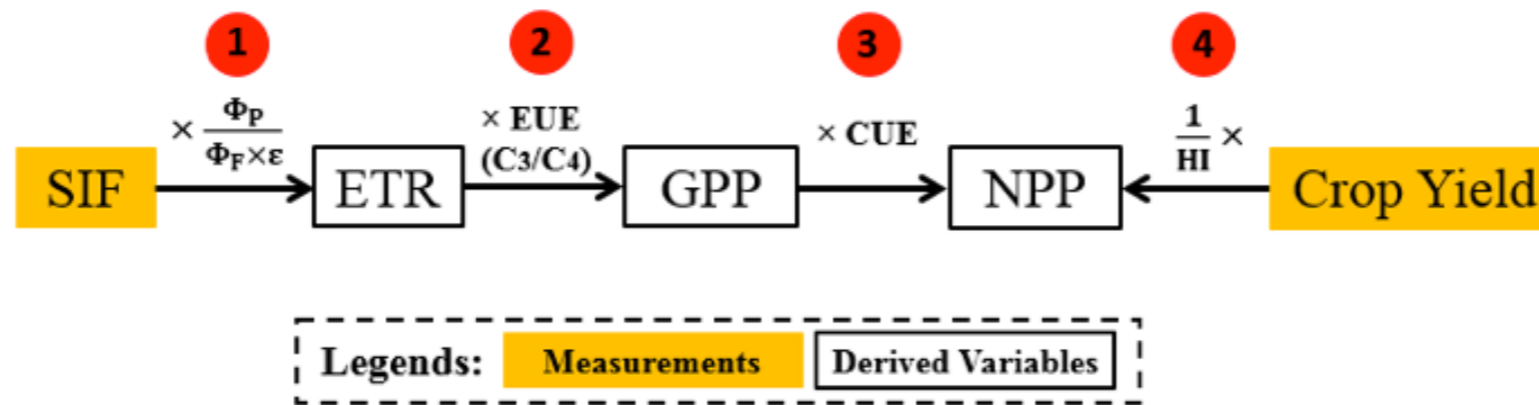




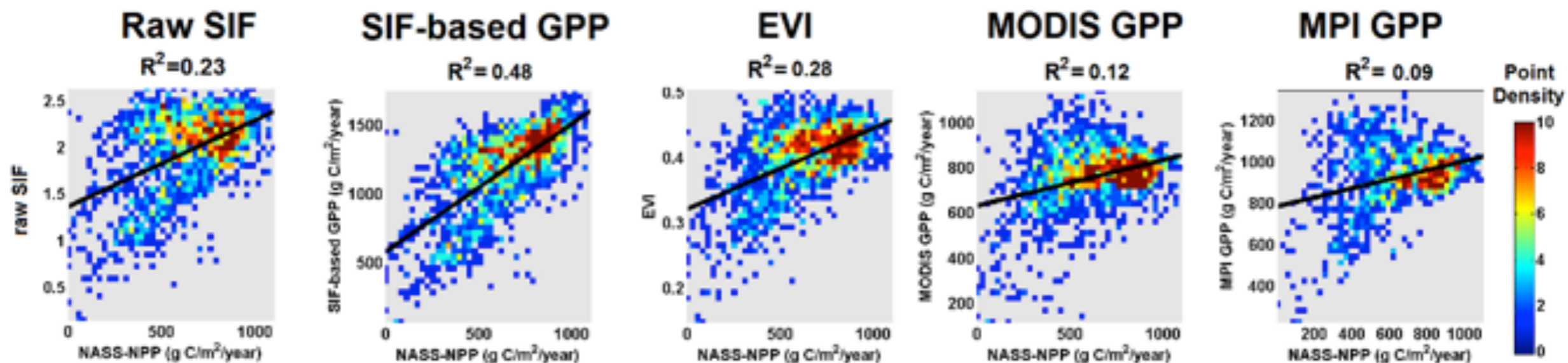
# THE SCATTERING IMPACT, ADDED ADVANTAGE OF FLUORESCENCE (ESP. IN TROPICS)



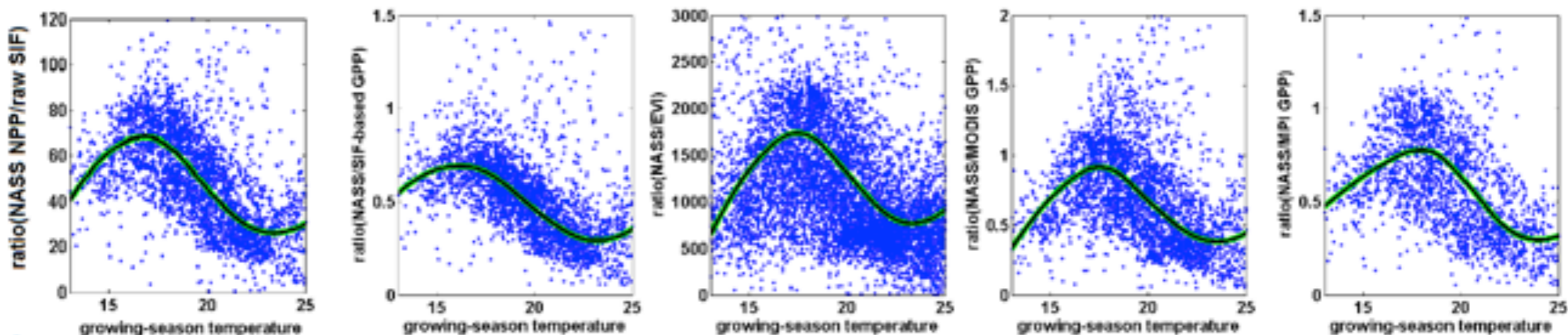
Frankenberg et al, AMT (2012)



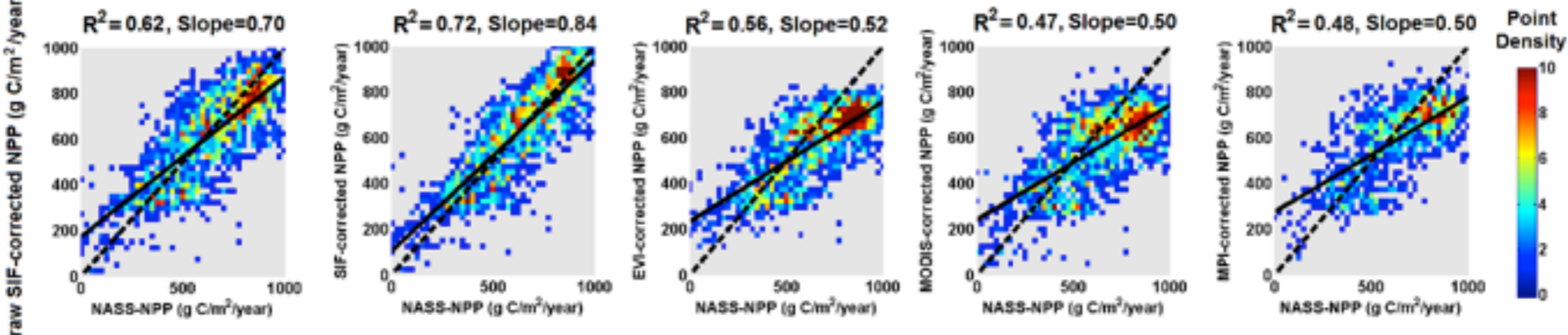
Raw Data



Apparent CUE



Calibrated NPP





$$(2 + 1.5\beta/\alpha) V_c \left[ V_{G_{max}} (1 - V_c/\alpha) + \frac{V_{N_{max}}}{1 + K_I/I} \right] = V_{G_{max}} (1 - V_c/\alpha)$$

$$V_c^2 \left( \frac{1.5\beta}{\alpha} - \frac{V_{G_{max}} (2 + 1.5\beta/\alpha)}{\alpha} \right) + V_c \left( (2 + 1.5\beta/\alpha) \left( \frac{V_{G_{max}}}{1 + K_I/I} + \frac{V_{N_{max}}}{\alpha} \right) + \frac{V_{G_{max}}}{\alpha} \right) - \frac{V_{G_{max}} V_{N_{max}}}{1 + K_I/I} = 0$$

$\times$  b.s by  $\frac{(1 + K_I/I)}{-V_{G_{max}}}$

$$\frac{V_c^2}{\alpha^2} (2\alpha + 1.5\beta)(1 + K_I/I) - \frac{V_c}{\alpha} \left[ (2\alpha + 1.5\beta)(1 + K_I/I) + \frac{V_{N_{max}}}{V_{G_{max}}} + \frac{V_{N_{max}}}{\alpha} \right] + \frac{V_{N_{max}}}{V_{G_{max}}} = 0$$

$$\frac{V_c^2}{\alpha^2} (2\alpha + 1.5\beta)(1 + K_I/I) - \frac{V_c}{\alpha} \left[ (2\alpha + 1.5\beta)(1 + K_I/I) + \frac{V_{N_{max}}}{V_{G_{max}}} + V_{N_{max}} \right] + V_{N_{max}} = 0$$

$$V_o = \frac{\beta}{\alpha} V_c$$

$$\frac{V_c}{\alpha} = \frac{\left[ (2\alpha + 1.5\beta)(1 + K_I/I) + \frac{V_{N_{max}}}{V_{G_{max}}} + V_{N_{max}} \right] \pm \sqrt{\left[ (2\alpha + 1.5\beta)(1 + K_I/I) + \frac{V_{N_{max}}}{V_{G_{max}}} + V_{N_{max}} \right]^2 - 4(2\alpha + 1.5\beta)(1 + K_I/I)V_{N_{max}}}}{2(2\alpha + 1.5\beta)(1 + K_I/I)}$$

1302  $I \equiv I, K_I \equiv I1, V_{N_{max}} \equiv N, V_{G_{max}} \equiv P$

$\alpha \equiv V_G, \beta \equiv V_\phi$

remember change 130, 140, 270

$V_c \equiv W_G, V_o \equiv W_\phi$

130 I = 2000  
 135 I1 = 1000  
 140 P = 430  
 145 N = 39

P 1629x

N 146x  
 6